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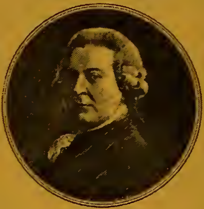

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A SHORT

COMMENT

ON

SIR I. NEWTON's

PRINCIPIA.

CONTAINING

NOTES upon ſome DIFFICULT PLACES

OF THAT

EXCELLENT BOOK.

Magnum iter aſcendo, —— PROPERT.

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Hon. Chas. F. Adams,
July 2, 1891.

THE

PREFACE.

THE Principia *being a book which is univerſally read by all the world, that pretend to any degree of philoſophical learning; it cannot be improper to explain ſuch paſſages therein as ſeem obſcure and difficult. For altho' it is written in as clear a ſtile as can be done in ſo few words; yet, by reaſon of its conciſeneſs, and the difficulty of the ſubjects treated on, many things occur which require ſome farther explication, eſpecially to young beginners.*

Accordingly ſeveral mathematical writers have endeavoured to explain ſome parts or other, of this book, to make them intelligible to common readers, who, without ſuch helps, would find it very difficult to get forward.

The noble ſubjects this book treats of, being no leſs than the grand fabric of the world, and the whole ſyſtem of nature, although compriſed in ſo little a compaſs, makes it highly deſerving of every illuſtration that can be given it.

The author has clearly ſhewn in this book, that all the bodies in the world are actuated by the univerſal principle of Gravity; which is this, that every body is attracted or impelled towards any other body, by a force which is reciprocally as the ſquare of the diſtance of the two bodies. On this univerſal principle he ſhews, that the motions of all the great bodies in the world are founded.

Several men had written ſyſtems of Philoſophy before Sir Iſaac; *but from their ignorance of nature, none of them could ſtand the teſt. But his Principles being*

built

built upon the unerring foundation of Obſervations and Experiments, muſt neceſſarily ſtand good, till the diſſolution of nature itſelf.

This little Treatiſe was written many years ſince; for when I ſtudied the Principia, I was frequently at a ſtop, which obliged me to make calculations here and there, as I went on; and when I had done, I ſet them down as notes upon theſe places. Wherein I only meddled with theſe places, that appeared difficult to me. Theſe notes collected together are the ſubject of the following Comment. And I have reviſed the whole, and added ſeveral things that ſeemed wanting. Yet I believe there are ſome things ſtill behind, which are not ſufficiently explained by any Commentator; and eſpecially ſuch as are there laid down without their demonſtrations.

As no body is reckoned a Philoſopher that does not read the Principia; therefore I thought proper to publiſh this ſmall Tract, ſuppoſing that it may be uſeful to others, that have a deſire to read that celebrated work. What I have further to mention, is this. The paſſages referred to, and which are to be explained, are not taken from the Latin edition, which would not ſuit the Engliſh reader; but from Motte's *tranſlation, and from the firſt edition thereof, if there happen to be more.*

W. Emerſon.

ADVERTISEMENT.

Beſides the common Algebraic characters which are in uſe, I make uſe of this $\propto$, which ſignifies a proportion. Thus A $\propto$ BC, ſignifies that A is in a conſtant ratio to BC, or that A is as BC.

A

SHORT COMMENT,

CONTAINING

Explanations on Sir ISAAC NEWTON'ſ Principia, as Tranſlated by AND. MOTTE.

[COR. 2. to the laws. And therefore if the weight p is to the weight A, &c.] For p will have the ſame effect as P, if $p : P :: pH \times OL : pN \times$ perpendicular from O on pN. And $P : A ::$ (by what went before) $KO : OL$. Therefore, *ex equo*, if p and A are in equilibrio, it will be $p : A :: pH \times KO : pN \times$ perpendicular from O on pN. Note, the line pN ought to be drawn in fig. 2, and not PL.

[Cor. 4. ib.] This is demonſtrated in Keil's Introduction, Theor. 20; as likewiſe in Lem. 23, p. 127, of this book.

[ib. p. 28, *ſub finem*; but the diſtance between theſe two centers,] that is, between the center of the two, and the center of all the reſt. Further, the actions of all the bodies may be conſidered as the ſum of the actions of every two; and then the caſe will be plain.

[Sch. to cor. 6, p. 33, then will ST repreſent] For if RV = retardation of deſcribing 2RA + 2VA, ST will be the retardation of deſcribing $\frac{1}{4}$ of that, or $\frac{1}{2}$ RV + $\frac{1}{2}$VA, that is SA or ~~F~~A; and T
therefore the body falling from S in the air, or from

Fig. T in vacuo; will have, nearly, the ſame velocity in A; the ſame of *s*A or *t*A; for aſcending to *s* in the air, or aſcending to *t* (or deſcending from *t* to A) in vacuo has the ſame velocity in A.

[ib. p. 38, but if they are turned aſide by the interpoſition] This is plain by Prop. 16, Mechanics.

[ib. And in like manner, &c.] By the ſame Prop. as before.

BOOK I.

SECT. I.

[LEM. 10.] For let AE be divided into an infinite number of equal parts, any part, as D′ × by the velocity acquired in the time AD (that is, D × DB), is as the ſpace deſcribed in that little part of time D′ (for the ſpaces are as the times × by the velocities); and the ſum of all theſe products or areas, that is, the areas ADB, AEC, are as the whole ſpaces deſcribed in the times AD, AE; but theſe areas are as AD^2 and AE^2, by Lem. 9.

[ib. cor. 1. — to the bodies, and meaſured —] that is, and the ſaid errors meaſured by the diſtances of the bodies, &c.

[Lem. 11. have a finite curvature] Theſe words exclude thoſe curves whoſe radius of curvature is infinitely ſmall, or infinitely great.

[ib. Schol.] All this may be univerſally demonſtrated after this manner. Let AB, AF be two paraboloids. Let, latus rectum, of AF $= a$, of AB $= b$. AC $= z$. AE $= x$. CB, EF, or AD $= y$. And let m be any affirmative index, and ſuppoſe $a^m x = y^{m+1}$. And $b^{m+n} z = y^{m+n+1}$. Then $a^{\frac{m}{m+1}} x^{\frac{1}{m+1}} = y = b^{\frac{m+n}{m+n+1}} \times z^{\frac{1}{m+n+1}}$. And by involution, $a^{m \times \overline{m+n+1}} x^{m+n+1} = b^{\overline{m+n} \times \overline{m+1}} z^{m+1}$. And therefore, $a^{m^2+mn+m} x^n : b^{m^2+mn+m+n} :: z^{m+1} : x^{m+1}$. But, becauſe x is infinitely ſmall, therefore when n is affirmative, the firſt term is infinitely leſs than the ſecond; and therefore z^{m+1} is infinitely leſſer than x^{m+1}; but if 1.

Fig. 1. n is negative, the firſt term is infinitely greater than the ſecond, and therefore z^{m+1} is infinitely greater than x^{m+1}, or z than x; that is, in caſe the firſt DB is infinitely leſs, and in caſe the ſecond, infinitely greater than DF. But DB, DF are as the curvatures of AB, AF, therefore, &c.

SECT. II.

2. [Pr. 1. cor. 4. as the verſed ſines, of arcs] deſcribed in equal times, tending to the center of force, and biſect the chords; let AB $=$ BC, and Bd be $\perp$ to AC, then Ad $=$ dC. And when the arch AC is diminiſhed to infinity, e coincides with d; and conſequently Ae $=$ eC, or BeS biſects the chord AC. Complete the parallelogram ABCf, and $e\mathrm{B} = \frac{1}{2}f\mathrm{B}$. Note, he calls that the verſed ſign of an arch, which is commonly called the verſed ſine of half that arch.

[Pr. 4, cor. 2.] For the forces $\propto \frac{\text{velo.}^2}{\text{rad.}} \propto \frac{\text{vel}^2 \times \text{rad}}{\text{rad}^2} \propto \frac{\text{rad}}{p\ \text{times}^2}$; all the other corollaries (except the laſt) depend on this.

[ib. cor. 8.] Let R $=$ radius of curvature D $=$ diſtance from the center of force. Then will the p.time $\propto R^n \propto D^n$ (by ſimilar poſition), and the areas $\propto$ velocities $\propto R^{n-1} \propto D^{n-1}$. And therefore the force $\propto \frac{1}{D^{2n-1}}$: and the contrary.

3. [ibid. cor. 9.] Take the arch Bd infinitely ſmall, and let BF be deſcribed by the revolving body, in the ſame time that a body falls from B to E by an uniform centripetal force, which it has at B; then Cd is the ſpace fallen in the time of deſcribing Bd. But Cd or Bn : BE : : (as the ſquares of the times,

that is, as) $Bd^2 : BF^2 :: \frac{Bd^2}{BA} : \frac{BF^2}{BA}$. But $Bn = \frac{Bd^2}{BA}$. Therefore $BE = \frac{BF^2}{BA}$. And $BE : BF : BA$ ∺. Fig. 3.

[ib. Schol. as the ſquare of the length applied to the radius;] for, the number of reflections is ∝ velocity or length directly, and the radius reciprocally.

[Pr. 7. cor. 2. in the ſame periodical time] Let ac be ‖ to RP, and da ‖ to PS. Then if the periodic times be equal, the areas generated in a given infinitely ſmall time muſt be equal, that is, the velocities round R and S muſt be reciprocally as (the ⊥'s on PG from R and S, that is, as) RP and SG. And ſuppoſing a, c, d, to coincide in P, the force round R to the force round S is in the complicate ratio (of ac to ad, or) SG to SP, and the ſquares of the times of deſcribing a given arch, that is reciprocally as the ſquares of the velocities, that is, as SG^2 to RP^2. Therefore the force round R to force round S : : is as SG^3 to $SP \times RP^2$, when the periodical times are equal. 4.

Or thus, let p be the place of the body when the tangent pg is ‖ to the line RS. Then the velocities round R and S, in the place p, will be equal; for the ſmall areas are equal, and their heights are equal, by reaſon of the parallels RS, pg. Draw TV, tv, and then by ſimilar triangles $Sp^2 \times pv^3 = Sg^3 \times pt^3$. And $Sp^3 \times PV^3 = SG^3 \times PT^3$. Then by this prop. force round S, in P : force round S, in $p :: Sp^2 \times pv^3$ or $\frac{Sg^3 \times pt^3}{Sp} : SP^2 \times PV^3$ or $\frac{SG^3 \times PT^3}{SP}$.

And force round S, in p : force round R, in p : : $Sp : Rp$ or Sg.

Alſo

Fig. Alſo force round R, in p : force round R, in p : :
4. $RP^2 \times PT^3 : Rp^2 \times pt^3$ or $Sg^2 \times pt^3$: : *ex equo*, force
round S, in P : force round R, in P : : $RP^2 \times PT^3 : \frac{SG^3 \times PT^3}{SP} :: SP \times RP^2 : SG^3$.

[Cor. 3, in the ſame periodic time] for then the infinitely ſmall and equal areas will be deſcribed in equal times in P, and both theſe areas and the forces will be the ſame, as in a circle of the ſame curvature with the orbit at P, and therefore the forces are the ſame as in the foregoing Corol.

5. [Pr. 8. Sch.] let ApD be an ellipſis, AP a circle. Then $rn : Rn :: (pm : Pm ::)\ qn : Qn$. And by diviſion, $qr : QR :: (qn : Qn ::)\ CD : CA$. The force in the ellipſis $\propto \frac{qt^2 \times Sp^2}{qr}$ reciprocally $=$ (becauſe $qr = \frac{QR \times CD}{CA}$) $\frac{QT^2 \times CA \times Sp^2}{QR \times CD} =$ $\left(\text{becauſe } \frac{QT^2}{QR} = \frac{2Pm^3}{CA^2}\right) \frac{2Pm^3 \times Sp^2}{CA \times CD} =$ (becauſe $Pm^3 = \frac{pm^3 \times CA^3}{CD^3}$) $\frac{2pm^3 \times CA^2 \times Sp^2}{CD^4}$. Therefore (becauſe CA, Sp and CD are given) the force in the ellipſis is $\propto pm^3$ reciprocally. But in the hyperbola and parabola (where CA is negative or infinite) theſe lines are ſtill given, and therefore the force in any conic ſection is reciprocally as mp^3.

[Pr. 9. will be changed—] that is $\frac{QT^2}{QR}$ is every where the ſame ratio, viz. $\propto$ SP.

[ib. ſecond way.] for PV is (by reaſon of the given angle at P) as the radius of curvature, that is (by reaſon of the ſimilarity of the parts of the figure PQ) as SP.

[Pr. 10. ſecond way. *Add* the rectangle nPv.] for $Qv^2 + nPv = Qv^2 + \overline{TP + Tv} \times \overline{TP - Tv}$
$= Qv$

$= Qv^2 + TP^2 - Tv^2 = QT^2 + TP^2 =$ ſquare of Fig. the chord QP ; and $Pv \times nV + nPv = VPv$. 5.

And if the line QV be drawn, and a circle thro' the points PQV; the triangles PQv, and PQV will be ſimilar (the $\angle$QVP being = QPR = PQv), and therefore $Pv : PQ : PV$ ∺.

[Pr. 10. Sch. in the ratio of the diſtances from the center] for the fluxion of the ordinates is augmented or diminiſhed in the ſame ratio, and that is as the force.

SECT. III.

[Pr. 13, cor. 2.] For, in the demonſtrations of Prop. 11, 12, and 13, QT^2 is always equal to $QR \times$ latus rectum.

[Pr. 16, cor.] The four firſt corollaries are general, and agree to all conic ſections ; the ſixth corollary belongs to one and the ſame parabola.

[ib. cor. 6, it is more variable] that is, in the ellipſis, the ratio of the velocity at a leſs and greater diſtance, is greater than the ratio of the ſquare roots of the greater and leſſer diſtance; in the hyperbola 'tis greater; for the ratio of the greater and leſſer perpendiculars (which is the ſame with this ratio of the velocities) is greater than the ratio of the ſquare roots of the greater and leſſer diſtances in the ellipſis, and leſſer than it in the hyperbola. For in the ellipſis the perpendiculars in the greateſt and leaſt diſtances are the ſame with theſe diſtances; and in the hyperbola, the greateſt perpendicular poſſible is that from the focus on the aſſymptote, and the leaſt, the diſtance to the vertex. Alſo (by Conics) the perpendicular $Sy \propto \sqrt{\frac{SP}{HP}}$ (fig. 3.) See Clark. (alſo cor. 7.)

[Pr. 17. yet greater velocity] then PH (and PK) will be negative.

Fig. [ib. cor. 2.] The velocity in a circle is = ve-
5. locity acquired by falling thro' $\frac{1}{2}DS$, by the given
centripetal force.

SECT. IV.

[Pr. 18, 19, 20.] two given points or right lines.

[Pr. 20, caſe 4. But becauſe of the ſimilar triangles VSH, vsh,] For (by ſimilar Δ's SVP, shq) $SV : SP :: sh : sq ::$ (by ſim. Δ's, svh, spq) $sv : sp$. And (by the ſimilar Δ's SHP, shp) $SP : SH :: sp : sh$; ergo $SV : SH :: sv : sh$; and the angle $VSH = psq = vsh$.

[Pr. 21. three given points or right lines.] If three tangents be given, you have 3 points Y, from which 3 equal right lines as YH are to be drawn to H, by caſe 3d of the laſt Lemma. If 2 tangents and a point P, there will be given two points Y, from which two equal lines are to be drawn to the focus H, and a third point P, from which PH is to be drawn; by caſe 2d Lem. In the hyperbola, it is $PH - YH = SP$. If three points P be given, it is done by caſe 1 Lem.

SECT. V.

[Lem. 17. in given angles] each to each reſpectively is the ſame invariable angle, tho' they are not all equal.

[Lem. 18. and ſo (by ſuppoſition)—] for p and b are in the curve, and the rectangles of the lines from p and P are in the given ratio.

[ib. Sch. if p happens to be in a right line]
For if $rpq : spt :: RPQ : SPT$. And P be placed
in pB; then by ſimilar triangles $pq : PQ :: (pB :$
6. $PB ::) pt : PT$. And $rp : RP :: (Cp : CP ::) ps :$
PS. And by multiplying you have $rpq : RPQ ::$
$spt : SPT$; which agrees with the Lemma. But
if P is placed out of pB, as at n; then becauſe the
ratio

ratio $\frac{RnQ}{SnT}$ is greater than $\frac{RPQ}{SPT}$ it is alſo greater Fig. 6.
than $\frac{rpq}{spt}$, which is againſt the hypotheſis.

If p is placed in DC, then rpq will be $= 0$, and therefore $RPQ = 0$; and P will alſo be placed in DC..

[ib. If the two oppoſite angles] The Δ's PCR, PBT are ſimilar, for the angles at S and T are right, and $C = B$, being on the ſame arch PD. Therefore $PR : PC :: PT : PB$. And the triangles PBQ, and SCP are ſimilar, for the angles at S,Q 7.
are right, and $SCP = PBA$. Therefore $PC : PS :: PB : PQ$; and, *ex equo*, $PR \times PQ = PS \times PT$: and, *e contra*, if $PR \times PQ = PS \times PT$; the locus of the point P is a circle.

And if theſe lines are not perpendicular, yet ſince their lengths will be reciprocally as the ſines of the angles, it will follow that $PQ \times PR : PS \times T ::$ ſine of $S \times S$ of T : ſine of $Q \times$ ſine of R, and the contrary.

[ib. and one or two] If the point B be ſuppoſed 8.
to move towards A and at laſt to coincide with it, ſo that AB become a tangent at B, the Lemma will ſtill hold. And if B paſs beyond A, then the figure will be converted into this BECDEAB; the Lemma will ſtill hold as before. And if B move to an infinite diſtance, then DB, AB will be parallels; and C alſo, then DC, AC will alſo be parallel; and alſo the conic ſection paſſing thro' A and D, will paſs on infinitely towards C and B.

[Lem. 19.] This may be reſolved as Prob. 12, in the Univerſal Arithmetic.

[Lem. 20.] Here is another figure relating to this Lemma (fig. 9.)
9.

[Lem. 21. Therefore (by Lem. 20) the point D] for the angles CPR, BPT, CPB are given, and therefore the lines PT, PR (to which the ſides AQ, Fig. AQ, AS of the parallelogram are parallel) are given by poſition (ſee fig. above).

10. Here is another figure of Lem. 21. (fig. 10) This Lemma is the ſame with Prob. 53, of the Univerſal Arithmetic.

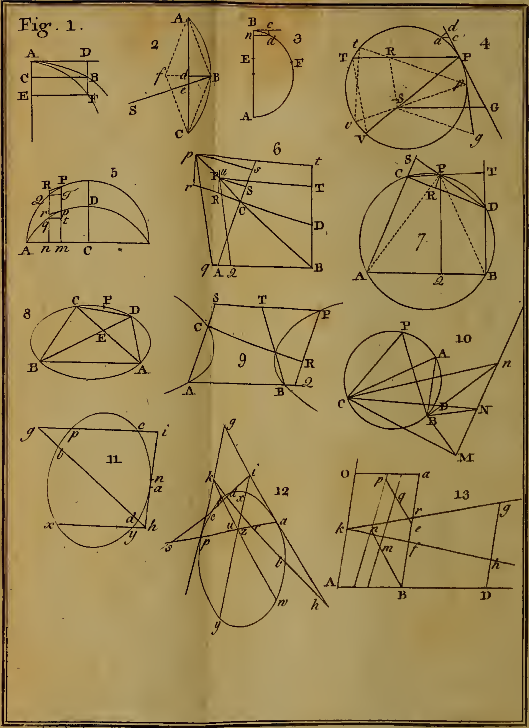
11. [Prop. 22.] This is the ſame with Prob. 55, in the author's Univerſal Arithmetic.

[Pr. 23. caſe 1.] This is the ſame with Prob. 56, of the Univerſal Arithmetic, or Prob. 57.

[Pr. 23. caſe 2. HA^2 will become to AI^2] for let n be a point in the conic ſection infinitely near a, thro' both which the line ih paſſes; then ih is a tangent at a. Then (Cor. 4. Pr. 44. B. I. my Conics) it will be, $bgd : bhd :: pgc : xhy = \frac{bhd \times pgc}{bgd}$, and hna or $ha^2 : ian$ or $ia^2 :: xhy$ or $\frac{bhd \times pgc}{bgd} : pic$ $:: bhd \times pgc : pic \times bgd$.

12. [Pr. 24. — will (by the properties of the Conic Sections)] becauſe any tangent may be ſuppoſed to cut the curve in two points infinitely near each other, therefore (by Cor. 4. Pr. 44. B. I. Ellipſis) theſe proportions follow. Alſo draw $kw \parallel$ to ga, to interſect pg, and let $nk^2 = tkw$. Then tkw or $nk^2 : ha^2 :: dkb : bhd :: kr^2 : hr^2$. And $nk : ha :: kr : hr$. And n, r, a, fall in one right line. Again, $kn^2 : ga^2 :: kp^2 : gp^2$. And $kn : gn :: kp : gp$. Whence n, p, a fall in one right line. And therefore the points p, r, a are in one right line.

13. [Lem. 22. Thus any right lines converging] Let the lines be kg, kh. Draw OB, OeD, and $pe \parallel$ to nB. Let m, q be the projected points of f and h. Then (becauſe in the point B, OB = OD,) mn (or Bn — Bm) $= fr$. Alſo qp (or $ep - eq$) : hg (or Dg — Dh) :: Oe : oD :: AB : AD :: fr or $mn : hg$. Therefore $qp = mn$, therefore pn, qm (which are the projected lines of gr, hf,) are parallel,



Principia

Pl. I. pa. 11.

Fig. 13. [ib. we ſhall have the ſolution required] For the figure *bgi* ſuppoſed now to be given, may be transformed into the firſt figure HGI, by making as O*d* to *dg*, ſo is OD to DG parallel to the radius AO.

[ib. For as often as two conic ſections] For theſe conic ſections being transformed into ſimpler ones, give the point of interſection; and thereby is had an ordinate drawn from that point of interſection in the transformed curves, correſponding to the interſection of the given curves.

14. [Pr. 25.] Let KG, KH be two tangents meeting in K, *yx* the third tangent meeting *ba* in *y*. Let the lines KH, KG be projected in *ki lb*; and *yb*, *yx* into *kl*, *ib*; then you have the parallelogram *bikl*; then proceed according to the propoſition.

[ib. For by the properties of the conic ſections] by cor. 4. Pr. 44. Conics I.

15. 16. [ib. But according as the points] This is plain, from the nature of the ellipſis and hyperbola; and the figure cannot be a parabola, by reaſon of two parallel tangents *ib* and *kl*.

17. [Lem. 23.] The Lem. is univerſal, as will appear by applying the demonſtration to this fig. 17.

[Lem. 24, from the nature of the conic ſections] by Pr. 46. B. I. of my Conic Sections.

18. [ib. cor. 1.] This holds as well when the tangent FG is on the other ſide.

[Lem. 25. alſo KH is to HL] by cor. 1. Lem. 24, for the tangents FH, LH cut the parallel tangents ML, IK in F and K.

19. [Lem. 25. cor. 2.] This holds as well when *qe* is on the other ſide of the figure; for, in all caſes (by cor. 1.) $KQ \times ME$ is given wherever the points Q, E fall, as ſuppoſe in *q*, *e*: for (by cor. 1. Lem. 24.) $Bq : AM$ or $BK :: eI : eM$. And by diviſion $Kq : BK :: MI : eM$. And $Kq \times eM = BK \times MI =$ (by the prop.) $KQ \times ME$; and $KQ : Me ::$ ($Kq : ME ::$) $Qq : Ee$.

[ib.

Fig. [ib. cor. 3.] for ſince $eM : ME :: QK : Kq$.
19. Therefore (by Lemma 23.) if the right lines eQ, MK, and Eq be drawn, the points of biſection will be placed in a right line given in poſition.

[Pr. 27. Sch. deſcribe the circle BKGC.] for the angle BKC (which is equal to the ſum of the given angles PBK and KCP) is always given.

[ib. and when thoſe other legs CK, BK] For it muſt be obſerved, that when the lines BP, CP, touch the curve at an infinite diſtance, that theſe lines are parallel to one another, and to the aſſymptote. Then to know the poſition of the aſſymptote; as the lines B*k*, C*k* revolve round the circle, the interſection *k* will ſometime fall into the line MN, as at the point N, then BN is parallel to one aſſymptote. For the interſection muſt neceſſarily fall in the line MN. And for the ſame reaſon the line BM will be parallel to the other aſſymptote. And therefore if the angle between them be biſected by the line OH (which is done by the perp. OH), then that line is the greater axis, or parallel to it.

[ib. Sch. But the ſquares of the axes] For the angle NBM = angle between the aſſymptotes = NLM, and NLH = half the angle of the aſſymptotes. Therefore LH is to HN, as the tranſverſe to the conjugate. And the ſquares of theſe axes, are as LH^2 to HN^2, or as HN^2 to HK^2, that is, as LH to HK.

20. [ib. There are alſo other Lemma's] For if the ſections are ſimilar, and in ſimilar poſition, and concentric; the tangent *acb* in *c*, is parallel to the tangent *dxe* in *x*, and therefore *ab* the ordinate is biſected in *c* the point of contact.

[Lem. 26. cor.] This is the ſame with Prob. 32, of the Univerſal Arithmetic.

[Lem. 27. cor. in the conſtruction] for then $IH : HF :: (IX : XY ::)\ ih : hf$. And $IH : HG ::$
(iL :

Fig. 20. (iL : LM : :) ib : hg. Or, if it be made as iL : : LM : : IH : HG, it will be ib : hf : : IH : HF; therefore on the contrary, if it be ib : hf : : (iE : EV : : iX : XY : :) IH : HF. It will be iL : LM : : IH : HG, which comes to the former conſtruction. For the ſolution of Prop. 22, 23, 24, 25, 26, 27; ſee Prop. 70, 71, 72, 73, 74, 75, B. III. my Conic Sections.

21. [Pr. 29. Sch.] Make alſo KA to AS, and LT to AT, as HG to GF, and draw MS, NA. Then the figures SAKM and ATLN are ſimilar to FGHI, and ſince three of the angles S, A, K or A, T, L are in the proper lines CB, ED, DB; if the fourth angle M or N, was in the fourth line EC, the problem would be rightly conſtructed. Therefore it is plain, its place can be no where, but where the line MN interſects EC as at i, which is the place of the angle I.

Now we are to prove that PQ cuts BA in f where F is to be placed. The triangle FGI is ſimilar to PEi (by conſtruction), and ſuppoſe them ſimilar to fgi; then the triangles Pfi and Egi are alſo ſimilar; for the angles at i are equal, and the ſides, about theſe angles proportional; therefore the angle Egi = Pfi, and ſince goi = Qof, oQf or PQE will be = fig. So that to have fig ſimilar to FIG, PQF muſt interſect Eg in Q, to make the < PQE = FIG, and the reſt follows of courſe.

SECT. VI.

[Pr. 30, Cor. 1.] For the times are as the areas, that is, as $\frac{4}{3}$GH × AS to $\frac{2}{3}$ × AS × 2AS.

[ib. Cor. 2.] For wherever the point P falls, viz. infinitely near A, ſtill $\frac{4}{3}$GH × AS (= area APS) = $\frac{1}{2}$AP × AS. And 8GH = 3AP. Therefore GH or the velocity of H : AP or the velocity of P : : 3 : 8. But the velocity of H is every where the

Fig. the ſame, for always $\frac{4}{3}GH \times AS =$ area ASP. And $GH \propto ASP \propto$ time of deſcribing AP.

[ib. Cor. 3.] For AP is the chord of a circle paſſing thro' A, S, P; and whoſe center is H.

[Pr. 31. as GK the difference] For when F comes to touch the line GH, the point A will be diſtant towards G, from the line FQO (which will then be ⊥ to GH) by the ſine of AOR or AOQ; and is then at L; therefore GK = arc GF — ſine of the arc AQ. And GK is as the time, and ſo it was in the conſtruction; therefore the point P is rightly found.

22
23. [ib. Sch. but ſince] This is demonſtrated in Keil's Aſtron. lectures, pa. 289, 297; or thus. Let AON = N. Since (AB : SH : :) OQ : OS : : 57.29578 : B. Therefore B = OS in degrees of the circle AQ. And ſince (rad : ſine of AOQ : :) SO : SF : : B : D. Then will SF = D in degrees. Let q be the true place of the body; Q the aſſumed place. Now ſince the time is as the area $SAQ = OAQ + SOQ = \overline{QA + SF} \times \frac{OA}{2}$; therefore the time is as $AQ \pm SF$. And therefore nearly as $AQ \pm D$, but accurately as $Aq \pm SE$. Take $N\varphi = D$. Now $OE : OQ : :$ (LE or $SE - SF$ or) $Nq \pm N\varphi$ or $q\varphi$ or $Q\varphi - qQ : Qq$. And $Q\varphi : Qq : : QE : OQ$, by compoſition, and becauſe $QE = OQ \pm OE$. But by conſtruction $OQ = \frac{OS \times L}{R}$. And rad. (R) : coſ. of AOQ : : SO : OF or OE, therefore $OE = \frac{SO \times \text{coſ. } AOQ}{R}$ Wherefore $Q\varphi : Qq : : (QE =)$ $\frac{OS \times L}{R} \pm \frac{SO \times \text{coſ. } AQ}{R} : \frac{OS \times L}{R} : : L \pm$ coſ. $AQ : L$. But $N - AOQ + D = (AN - AQ + N\varphi =)\ Q\varphi$. Therefore $Qq = E$, and $AOq = AOQ + QOq = AOQ + E$, nearly, and therefore

fore E is rightly found. And repeating the ſame Fig.
work with theſe new angles, there will be found 22.
the angles F, G, H, I. 23.

[ib. the area AIKP will be given] by Prop. 87. cor. 2. and Schol. my Conic Sections, B. II. Alſo AIKP = OPA, by cor. 1. Prop. 86. ib.

Alſo area deſcribed = ASQ = ASP + PSQ = ASP + PQ × ½SN (nearly) = A (by ſuppoſition) therefore A — ASP = ½SN × PQ; and $\text{PQ} = \dfrac{2\text{A} - 2\text{ASP}}{\text{SN}}$, nearly.

24. [ib. and by ſuch computations.—But the particular calculus] with the radius 1 and center H deſcribe the circle RSE; and draw SP, S*p*, HPT, H*pf*. Let fall the perpendiculars *pt*, *pu*, upon the lines HP, SP; which will be equal, becauſe the angles *p*P*t* (HPB), and *p*P*n*, are equal.

Let the arch RT = z. Sine TQ = s, coſ. HQ = x, SP = y, HP = v, AO or OB = a, SO or OH = n, OD = c. T = mean anomaly, l = latus rectum.

By ſimilar ſectors, HT (t) : HP (v) :: Tf ($\dot{z}$) : pt or $pn = v\dot{z}$. And the area $\dfrac{\text{SP} \times pu}{2} = \dfrac{vy\dot{z}}{2} =$ fluxion of BPS. But (cor. 2. Prop. 23, ellipſis) $vy = cc + \dfrac{nn}{cc} \times \text{PM}^2$, and (cor. 3. Prop. 72, ib.) HP or $v = \dfrac{cc}{a + nx}$, and rad (1) : PH $\left(\dfrac{cc}{a + nx}\right)$ :: S : PM $= \dfrac{ccs}{a + nx}$, therefore $vy = cc + \dfrac{nnccss}{\overline{a+nx}^{2}}$ and area SP$p = \frac{1}{2}cc\dot{z} + \dfrac{\frac{1}{2}nnccss\dot{z}}{aa + 2anx \;\&c.} = \frac{1}{2}cc\dot{z} + \dfrac{nnccss\dot{z}}{2aa} - \dfrac{n^3c^2s^2x\dot{z}}{a^3}$. But $s\dot{z} = -\dot{x}$, and $x\dot{z} = \dot{s}$,

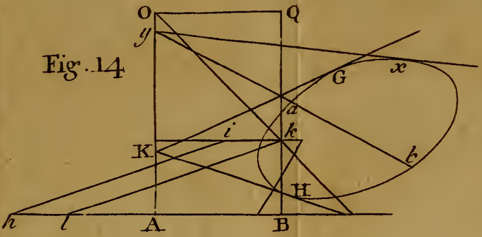
and

Fig. 24. and $\frac{cc}{a} = \frac{1}{2}l$; therefore the area SP$p$ $= \frac{1}{4}al\dot{z} - \frac{\frac{1}{4}nnls\dot{x}}{a} - \frac{n^3lss\dot{s}}{2aa}$, and the fluent BSP $= \frac{alz}{4} - \frac{nnl}{4a} \times$ area EHQT $- \frac{n^3ls^3}{6aa}$; and corrected, BSP $= \frac{alz}{4} + \frac{nnl}{4a} \times$ area RTQ $- \frac{n^3ls^3}{6aa}$. But RTQ $= \frac{1}{2}z - \frac{1}{2}xs$. Whence BSP $= \frac{1}{4}alz + \frac{\frac{1}{4}nnl}{a} \times \overline{\tfrac{1}{2}z - \tfrac{1}{2}xs} - \frac{n^3ls^3}{6aa} = \frac{2aa+nn}{8a}lz - \frac{nnls}{aa} \times \overline{\frac{ax}{8} + \frac{nss}{6}}$. And dividing, $z - \frac{8nns}{2a^3+ann} \times \overline{\frac{ax}{8} + \frac{nss}{6}} = T$, the mean anomaly. Whence $z = T + \frac{nn}{4aa+2nn} \times S.2T + \frac{4n^3}{6a^3+3ann} \times \overline{S.T}^3$, becauſe T is nearly $= z$, and $2xs = S.2T$. But ſince n is very ſmall by ſuppoſition, $z = T + \frac{nn}{4aa} \times S.2T + \frac{2n^3}{3a^3} \times \overline{S.T}^3$. Where the quantities $\frac{nn}{4aa} \times S.2T$, and $\frac{2n^3}{3a^3} \times \overline{S.T}^3$ are ſmall arches to be added to T.

Now $D = c - \frac{cc}{a} = \frac{c}{a} \times \overline{a-c}$; and $D \times \overline{AO + OD} = \frac{c}{a} \times \overline{aa - cc} = \frac{cnn}{a} = nn$ nearly, becauſe $c = a$ very near. Therefore $s.Y$: rad (1) :: $nn : 4aa$, and $s.Y$ or Y (in ſmall arches) $= \frac{nn}{4aa}$; alſo V : Y :: S T : rad (1), and V $=$ Y $\times$

S.2T

Fig. 14



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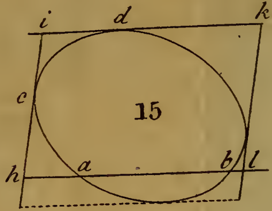

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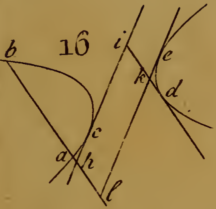

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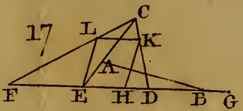

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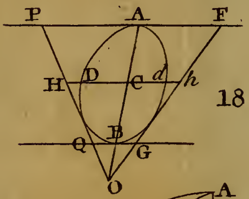

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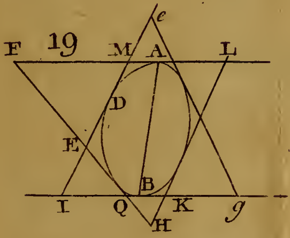

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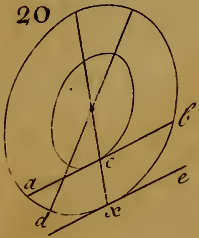

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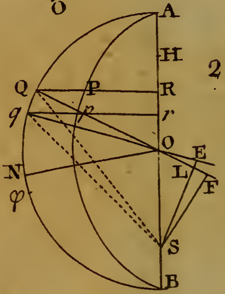

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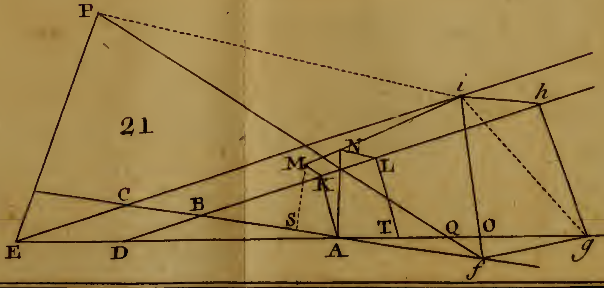

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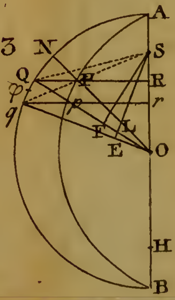


$\text{S.2T} = \frac{nn}{4aa} \times \text{S.2T}$, which is our firſt term, or his firſt equation. Fig. 24.

Again, $\text{S.Z} : \text{rad}\ (1) :: 4nD$ or $\frac{4nc}{a} \times \overline{a-c} :$ 3AO^2 or $3aa$, and $\text{S.Z} = \frac{4nc}{3a^3} \times \overline{a-c}$. But $\overline{a+c} \times \overline{a-c} = aa - cc = nn$, and $a - c = \frac{nn}{a+c} = \frac{nn}{2a}$ nearly, therefore $\text{S.Z} = \frac{4nc}{3a^3} \times \frac{nn}{2a} = \frac{2n^3c}{3a^4} = \frac{2n^3}{3a^3}$, nearly. Alſo $\text{X} : \text{Z} :: \overline{\text{S.T}}^3 : \text{rad}\ (1)$. And $\text{X} = \text{Z} \times \overline{\text{S.T}}^3 = \frac{2n^3}{3a^3} \times \overline{\text{S.T}}^3$, which is the ſecond term, or his ſecond equation. And when T is above 90°, then S.2T, and V will be negative, and $\angle\text{BHP} = \text{T} + \text{X} \pm \text{V}$.

He calls, Y or $\frac{nn}{4aa}$ the greateſt firſt equation, becauſe it is greateſt when $\text{S.2T} = 1$, and $\text{T} = 45°$. And Z or $\frac{2n^3}{3a^3}$ the greateſt ſecond equation, becauſe the greateſt it can be is when $\overline{\text{S.T}}^3 = 1$, or $\text{T} = 90°$.

SECT. VII.

[Pr. 33, to AB the principal ſemi-diameter] this ſhould be $\frac{1}{2}$AB.

[ib. cor. 2.] for the two firſt terms of the prop. are in a ratio of equality, and ſo the two laſt.

[Pr. 34. For (by cor. 7. prop. 16.)] and cor. 6. prop. 4.

[Pr. 35. The ſame things ſuppoſed] to wit, that the ſpace CS is as the area SDE of the circle, rectangled hyperbola, or parabola.

Fig. [Prop. 37, as appears by prop. 34.] and cor. 7.
25. pr. 16.

[Pr. 38, acquire the velocity CD.] by prop. 10. cor. 2. The periodical times of the ellipſis AD and AP are equal. The time of deſcribing AP is (as APS, that is, as ADS or) as AD. Let the ellipſis AP coincide with AC, and the time of deſcribing AC will ſtill be as AD. Further, draw *cd* ∥ to CD; becauſe the time of deſcribing the whole ellipſes AD*d*, AP*p* are equal; therefore in equal times they deſcribe areas which are as the whole ellipſes, that is, as CD to CP, or ADS to APS. Wherefore in the time D deſcribe AD or D*d*, P deſcribes AP or P*p*, and (when P coincides with C), C deſcribes AC or C*c*; therefore the velocity of D : to velocity of C : : is as D*d* : C*c* : : or as SD to CD; but SD and the velocity of D is given; therefore the velocity of C is as CD.

[Pr. 39. cor. 2, 3.] In theſe cor. the line PD is the ſpace the body would aſcend to, or fall from, (to acquire the velocity it is projected with) by a uniform centripetal force, according to cor. 1.

SECT. VIII.

[Pr. 40. cor. 1.] This is evident, by ſuppoſing ITK convex towards C.

26. [ib. cor. 2.] for by fluxions, let AC $=$ P. CD $=$ A $= x$. Then DG $\propto x^{n-1}$ (by hyp. and pr. 39.) and DEFG $=$ fluxion of the area $\propto x^{n-1}\dot{x}$. And the area DC*b*G $\propto \frac{x^n}{n}$. And when x becomes $= p$, the area AC*b*B $\propto \frac{p^n}{n}$. Therefore ADGB $\propto \frac{p^n - x^n}{n}$. And therefore by prop. 39, the velocity

in D, at the diſtance A or x, is $\propto \sqrt{\frac{p^n - x_n}{n}}$, or Fig. 26.
(becauſe n is given, and $x = \mathrm{A}$) as $\sqrt{\mathrm{P}^n - \mathrm{A}^n}$.

N. B. If the force is reciprocally as the diſtance, the curve BGb will be a rectangled hyperbola, to the aſſymptotes AC, Cb; for DG $\propto \frac{1}{\mathrm{CD}} =$ (ſuppoſe to) $\frac{bb}{\mathrm{CD}}$. And DG × CD $= bb$, which is the known property of the hyperbola. And the velocity at C will be infinite, for the area ABbC is infinite. And the velocity at any place D is as the hyperbolic area ABGD, which may be found by Stone's Fluxions, p. 54; or by Cor. 2. Prop. 87. B. II. my Conic Sections.

[Pr. 41. — in the Trajectories found.] One being given to find the other.

[ib. given the circle VR] that is, CV is known.

[ib. in the leaſt given time,] theſe ſmall parts of time are taken conſtant and invariable.

[ib. and the triangle ICK] that is, the triangle will be invariable.

[ib. and ſuppoſe the magnitude of Q,] This quantity is any conſtant quantity, but unknown. Suppoſe in ſome caſe $\sqrt{\mathrm{ABFD}} = b \times \mathrm{IK}$, and Z $= b \times \mathrm{KN}$. And in all caſes (by pr. 40.) $\sqrt{\mathrm{ABFD}} = b \times \mathrm{IK}$; and in all caſes Z or $\frac{\mathrm{Q}}{\mathrm{A}} = b \times \mathrm{KN}$, that is in all caſes Q $= b \times \mathrm{A} \times \mathrm{KN}$; which is plain, becauſe Q and A × KN are conſtant quantities, therefore if it be once $\sqrt{\mathrm{ABFD}}$: Z : : KI : KN, or $b \times \mathrm{IK} : b \times \mathrm{KN}$: : KI : KN; it will always be ſo.

In this prop. the line CE is indetermined; and if Q were known, the areas of the curves abz, and acx might be known to any diſtance CE.

Fig. [ib. cor. 3.] In fig. 4. Newt. Let CA $= a$ CV $= r$, CD $= x$, DF $= y$, $\sqrt{aa - rr} = n$.

Then the force DF being as $\frac{1}{x^3}$, the fluxion of the area ABFD is $\frac{-\dot{x}}{x^3}$, and the fluent as $\frac{1}{2x^2}$, and correɗted, the fluent is $= \frac{aa - xx}{2aaxx} =$ area ABFD. Therefore the velocity at D is as $\frac{\sqrt{aa-xx}}{ax}$. Then;

1. Suppoſe a infinite, and $x = r$, then the area $\sqrt{\overline{ABFD}}$ becomes $\frac{\sqrt{aa}}{ar} = \frac{1}{r}$. And ſince at V, the orbit IV is perp. to CV, therefore IK $=$ KN, and $\sqrt{ABFD} = Z = \frac{Q}{x}$, that is, $\frac{1}{r} = \frac{Q}{r}$, and $Q = 1$, and $Z = \frac{1}{x}$; alſo $\sqrt{\overline{ABFD}} = \frac{\sqrt{aa-xx}}{ax} = \frac{a}{ax} = \frac{1}{x}$ at any place I, and ſince $\sqrt{ABFD}$ $\left(\frac{1}{x}\right) : Z \left(\frac{1}{x}\right)$ :: IK : KN; therefore TK$=$KN, and IN $= o$; therefore in this caſe the orbit is a circle, as VXR.

2. If a be leſs than infinite, and ſince at V, the orbit is perp. to CV as before, therefore IK$=$KN, and $\sqrt{ABFD} = \frac{Q}{r}$, that is, $\frac{\sqrt{aa-rr}}{ar} = \frac{Q}{r}$, or $\frac{n}{ar} = \frac{Q}{r}$, and $Q = \frac{n}{a}$. Whence $Z = \frac{n}{ax}$. Therefore Db $= \frac{n}{a} \times \frac{1}{\sqrt{\frac{aa-xx}{aaxx} - \frac{nn}{aaxx}}} = \frac{n}{2a}\sqrt{\frac{aaxx}{rr-xx}} =$

nx

$\frac{nx}{2\sqrt{rr-xx}}$. And D$c = \frac{rr}{nx} \times$ D$b = \frac{nrr}{2x\sqrt{rr-xx}}$, Fig. 26.
and flux of the area V$ac$D $= \frac{nrr\dot{x}}{2x\sqrt{rr-xx}} =$ fluxion of the ſector VCX, and the Fl. : $\frac{nrr\dot{x}}{2x\sqrt{rr-xx}}$ $=$ Sector VCX, which is as the angle VCX.

But, putting ſemiconjugate $= c$, CH $=$ Z, the hyperbolic ſector VCR $=$ Fl. : $\frac{rc\dot{z}}{2\sqrt{zz-rr}}$, and CT $= \frac{rr}{z} = x$, and $z = \frac{rr}{x}$. Then inſtead of z and $\dot{z}$, putting their values; we ſhall have the hyp. ſector VCR $=$ Fl. : $\frac{-rrc\dot{x}}{2x\sqrt{rr-xx}} = \frac{c}{R} \times$ cir. ſector VCX Therefore when $x =$ CT, the hyp. ſector VCR : cir. ſector CVX : : $c : n$; that is, becauſe the angle VCX is as the ſector CVX, the angle VCX is to the hyper. ſector VCR in a given ratio. And ſince CT or x continually decreaſes, as the ſector VCR increaſes, the body P draws continually nearer the center C.

3. If the velocity be greater than falling from an infinite height; the flux area ABFD $= \frac{+\dot{x}}{x^3}$, and the fluent $\frac{-1}{2xx}$. But at firſt the area (ſuppoſe) $= \frac{bb}{2}$, and $x = a$ an infinite line; therefore ABFD $- \frac{bb}{2} = \frac{1}{2aa} - \frac{1}{2xx}$, and ABFD $= \frac{bb}{2} + \frac{xx-aa}{2aaxx}$. Therefore $\sqrt{}$ABFD is as $\sqrt{\frac{bbaaxx+xx-aa}{aaxx}}$ $=$(by ſubſtitution) $\sqrt{\frac{sxx-aa}{aaxx}}$.

Fig. 26. But in V, $x = r$, and $\sqrt{ABFD} = \sqrt{\frac{srr-aa}{aarr}} = \frac{Q}{r}$, and $Q = \frac{\sqrt{srr-aa}}{a}$, and $Z = \frac{Q}{x} = \frac{\sqrt{srr-aa}}{ax}$. Then $Db = \frac{\sqrt{srr-aa}}{2a\sqrt{\frac{sxx-aa}{aaxx}-\frac{srr-aa}{aaxx}}} = \sqrt{\frac{srr-aa}{s}} \times \frac{x}{2\sqrt{xx-rr}} =$ (by ſubſtitution) $\frac{Ax}{2\sqrt{xx-rr}}$, and $Dc = \frac{rr}{xx} \times Db = \frac{Arr}{2x\sqrt{xx-rr}}$; therefore $\frac{Arr\dot{x}}{2x\sqrt{xx-rr}} =$ flux. area V$ac$D, or of VCX. And the Fl. : $\frac{Arr\dot{x}}{2x\sqrt{xx-rr}} =$ ſector VCX, which is as the angle VCX.

27. But in the ellipſis VRS, let CH $= z$, ſemiconjugate $= c$. Then the elliptic ſector VCR $=$ Fl. : $\frac{rc\dot{z}}{2\sqrt{rr-zz}}$, and CT $= \frac{rr}{z} = x$, and $z = \frac{rr}{x}$, and $\dot{z} = \frac{-rr\dot{x}}{xx}$. Then putting for z and $\dot{z}$ their values, and the elip. ſector VCR $=$ Fl. : $\frac{-crr\dot{x}}{2x\sqrt{xx-rr}} = \frac{c}{A} \times$ cir. ſector VCX; therefore when $x =$ CT, the ellip. ſector VCR : cir. ſector VCX : : c : A; that is, becauſe the angle VCX is as the ſector VCX; the angle VCX is to the elliptic ſector VCR in a given ratio. And ſince CT or x continually increaſes, as the ſector VCR increaſes, the body P goes continually further and further from the center C.

[ib. and the Centripetal Force becoming centrifugal] for then the curve VPQ will turn upwards;

wards; and the law of the force being the ſame, Fig. 27. it will be conſtructed the ſame way by the elliptic ſectors; taking the point A between C and V. And here the velocity will increaſe, as it recedes from the center C. But it can never revolve round this center.

SECT. IX.

[Pr. 44. or in antecedentia with a celerity] for then the line mnC falls beyond s, sr being $= rk$, and $sCk = 2rCk$; and conſequently the point m falls without the circle.

[ib. and with a leſs force] if the orbit moves ſlower in antecedentia, than with twice the celerity of CP in conſequentia; for then the point m falls within the circle, between r and s.

The meaning of this Prop. is this, that the difference of the forces at different diſtances from the center, are reciprocally as the cubes of the diſtances (which forces are requiſite to make the body move in a quieſcent or revolving orbit; and the diſtances to be the ſame in both.)

[ib. cor. 1.] For mn repreſents the difference of the forces by which p revolves to n, or P to K in equal times; and the verſed ſine of RK repreſents the force whereby a body moves from R to K in the circle, in the ſame time.

[ib. cor. 2. as half the latus rectum] for theſe forces are as bn or ar to bp or as. But $ar = \frac{nr^2}{r\mathrm{D}} = \frac{ba^2}{a\mathrm{D}}$. And $as\,\mathrm{D} : sp^2 :: a\mathrm{D} : 2\mathrm{R}$; and $as\,\mathrm{D} = \frac{sp^2 \times a\mathrm{D}}{2\mathrm{R}} = \frac{ab^2 \times a\mathrm{D}}{2\mathrm{R}}$; and $as = \frac{ab^2 \times a\mathrm{D}}{s\mathrm{D} \times 2\mathrm{R}} = \frac{ab^2}{2\mathrm{R}}$. Whence theſe forces are as $\frac{ba^2}{a\mathrm{D}}$ to $\frac{ba^2}{2\mathrm{R}}$, or as R to aC or CV in fig. 2. pl. 18.

[ib.

Fig. 27. [ib. cor. 3.] For let the force in the immoveable ellipſis be $\frac{FFA}{T^3}$, and in V it will be $\frac{FFT}{T^3}$ or $\frac{FF}{TT}$. And the force by which a body may revolve in a circle at the diſtance CV is $\frac{FFR}{T^3}$. And the difference of the forces in V (of revolving in the moveable and immoveable ellipſis) is $\frac{GGR-FFR}{T^3}$, and in every altitude A is $\frac{GGR-FFR}{A^3}$; and the force in the moveable ellipſis will be $\frac{FFA}{T^3} + \frac{RGG-RFF}{A^3}$.

[ib. cor. 4.] After the ſame manner as in the two foregoing Corollaries; becauſe when the velocity is given, the force $\propto$ curvature or reciprocally $\propto$ radius of curvature; therefore T : R :: $\frac{VFF}{TT} : \frac{RVFF}{T^3}$, which is the force wherewith a body may revolve in a circle at the diſtance CV. And by cor. 1. FF : GG — FF :: $\frac{RVFF}{T^3} : \frac{RVGG-RVFF}{T^3}$, which is the difference of the forces (in the moveable and immoveable orbits) in the vertex, V. And by the prop. $\frac{1}{T^3} : \frac{1}{A^3} :: \frac{RVGG-RVFF}{T^3} : \frac{RVGG-RVFF}{A^3}$, which is the difference of forces in every altitude A; and therefore

fore the force in the moveable trajectory *upk* will 27.
be $X + \frac{RVGG-RVFF}{A^3}$.

[ib. cor. 6.] In this cor. $X = o$, and RVGG —
RVFF is compounded of all given quantities, and
the force in V*pk* will then be $\propto \frac{1}{A^3}$. This alſo ap-
pears from the Prop. for the difference of the forces
in P and *p* is as $\frac{1}{Cp^3}$, but the force in one of them,
viz. at P in the line VP is *o*; and therefore the
other force in V*pk* is the difference of the forces,
and is as $\frac{1}{Cp^3}$. Alſo this curve is the ſame with
that in cor. 3. Pr. 41. for let *vad* be a circle; and 28.
by this cor. 6. $\angle vcp \propto \angle vcz \propto$ arch *va* $\propto$ area
vca $\propto$ area *vcb*, which is the conſtruction in cor.
3. Pr. 41; alſo $cp = cz$ or *cx*, becauſe *vz*, *ax*, and
bx are tangents, which is alſo the ſame conſtruction
as in cor. 3. Pr. 41.

[Pr. 45, but orbits acquire the ſame figure] Let 29.
the ſmall parts of the curve *df*, *fh*, *hp*, &c. be
deſcribed by a body A in indefinitely ſmall given
parts of time. And let another body B go from
D in the ſame direction *de*, and with a velocity
which is to the velocity of the former in the ſub-
duplicate ratio of the centripetal force of B to that
of A; and let it arrive at *s* in the aforeſaid ſmall
given part of time. Now ſince *ef* : *rs* : : force of A :
force of B : : velocity^2 of A : vel.^{2} of B : : de^2 :
dr^2 : : ſquare of the time of B's moving thro' *de* :
ſquare of the time of B's moving through *dr*.
Therefore, by Lem. 10, the body B will paſs thro'
f. And ſince *rs* is every where as rd^2, it is evident
the ſame curve *dsf* will be deſcribed by both bodies,
and which *defg* touches in *d*, *f*. Alſo the velocity of A
in *f* : vel. of A in *d* : : as perpendicular from C or
de :

Fig. de : to ⊥ on fg : : vel. of B in f : vel. of B in d.
29. And vel. of A in f : velocity of B in f : : vel. A in d : vel. of B in D : : $de : dr$; that is in a given ratio; that is fg : to fk, and hn to hl, and px to pt are in a given ratio; and therefore when A is arrived at g, B would be at k, and ſince $gh : ki :: gf^2 : kf^2 :: ed^2 : rd^2 :: ef : rs$; or $gh : ef :: ki : rs$; that is, the centripetal of A to that of B at equal diſtances being in a given ratio, B will paſs thro' i, and conſequently thro' h. After the ſame manner it may be proved, that the body B will paſs thro' the points m, p, u, y, and deſcribe the ſame curve with the body A.

Further, if another body D move thro' Br ∥ to de, and with a velocity which is to the velocity of A as CB to Cd, and be acted upon by a force which in the points β, δ, z, &c. is to the force in the points d, f, h, &c. reſpectively, as $C\beta$ to Cd, $C\delta$ to Cf, Cz to Ch, &c. Then I ſay the curve deſcribed by the body D, viz. $\beta\delta z\varphi\omega$ will be ſimilar to the curve $dfhpy$; for in the time that A would arrive at e, D would arrive at γ; and becauſe $C\gamma : Ce :: C\beta : Cd ::$ force at β : force at d : : $\gamma\delta : ef :: Cr - \gamma\delta$ or $C\delta : Ce - ef$ or Cf; and $C\beta : C\delta :: Cd : Cf$; $\gamma\delta$ is every where as $\gamma\beta^2$ in all the points between β and γ (by Lem. 10.) it is manifeſt the fig. $C\beta\delta$ is ſimilar to the fig. Cdf; and the tangent $\delta\varepsilon$ ∥ to fg; and becauſe the areas $Cgf : Cfd :: C\varepsilon\gamma : C\delta\beta$; wherefore when A comes to g, D comes to ε; but $gh : \varepsilon z :: Cf : C\delta :: Cg : C\varepsilon :: Ch : Cz$, as before; and $Cf : Ch :: C\delta : Cz$, whence the fig. $C\delta z$ is ſimilar to Cfh; and after the ſame manner it will be proved, that the naſcent figures $Cz\varphi$, $C\varphi\omega$, are ſimilar to Chp, Cpy; and therefore the whole figure $\beta\delta z\varphi\omega$ is ſimilar to the figure $dfhpy$.

Or univerſally, if the orbits $\beta\delta\varphi$, and dfp are ſimilar. $C\beta \propto \gamma\delta \propto$ force × time^2 of deſcribing $\beta\delta \propto \text{force} \times \frac{\beta\delta^2}{\text{velocity}^2} \propto \text{force} \times \frac{C\beta^2}{\text{velocity}^2}$. Fig. 30. Therefore $\text{velocity}^2 \propto \text{force} \times$ diſtance $C\beta$. Whence if $C\beta$ be given, velocity $\propto \sqrt{\text{force}}$; and if velocity $\propto C\beta$, force $\propto C\beta$ alſo, which agrees with what went before. If the force be given, velocity $\propto \sqrt{C\beta}$. If velocity be given, force $\propto \frac{1}{C\beta}$.

[ib. Exam. 1. l. 3.] here $\frac{T^3 - 3TTX + 3TX^2 - X^3}{A^3}$ is twice repeated in the Engliſh, which is wrong.

[ib. By this collation of the terms] for ſince the ſame figure will be deſcribed by making the centripetal force proportional at equal diſtances (altering the velocity in the ſubduplicate ratio of the force); and ſince, when R is nearly $= T$; and x very ſmall, the centripetal force in the revolving ellipſis will be as $RGG - FFx$, and in this orbit (becauſe in both, the denominator A^3 is the ſame) is as $T^3 - 3TTx$, therefore theſe forces ought to be proportional at all diſtances, viz. when $x = 0$, and any other indeterminate diſtance A or $T - x$; wherefore (in theſe two caſes) $RGG : T^3 :: RGG - FFx : T^3 - 3TTx ::$ (and by diviſion) $FFx : {}^3TTx :: FF : {}^3TT$, which is the conſtruction of the problem. After the ſame manner in example 2d, the centripetal force in the revolving ellipſis, and this new orbit, will be as $RGG - FFx$, and $T^n - nT^{n-1}x$, and to have the orbits ſimilar, putting $x = 0$, $RGG : T^n ::$ (putting $T - x$ for any indeterminate diſtance) $RGG - FFx : T^n - nT^{n-1}x ::$ (by diviſion) $FFx : nT^{n-1}x :: FF : nT^{n-1}$. After the ſame manner in Examp. 3, it will be $RGG : bT^m + cT^n :: (RGG - FFx : bT^m + cT^n - mbT^{m-1}x - ncT^{n-1}x) :: FFx : mbT^{m-1}x + ncT^{n-1}x$. Or $GG : bT^{m-1} + cT^{n-1}$

::

Fig. : : FF : $mbT^{m-1} + ncT^{n-1}$. The quantity $\frac{FF}{AA} + \frac{RGG-RFF}{A^3}$ (in cor. 2. pr. 44.) is univerſally as the centripetal force, whether the apſides move backwards or forwards, for G : F : : VC*p* : VCP : : (p. 180.) *p*C*n* : *p*C*k*. And the force in the revolving ellipſis is greater than in the immoveable one, when *n* (fig. 2.) is without the line *ks*; and in that caſe (F is leſs than G, and) $+ \frac{RGG-RFF}{A^3}$ will be affirmative. But the force is leſs when *n* falls between *k* and *s*, for then (F is greater than G, and) $+ \frac{RGG-RFF}{A^3}$ is negative, as it ought to be.

SECT. X.

[Pr. 50. it is evident from the conſtruction] Since VP touches the curve in P, and PB is $\perp$ thereto, therefore B is the point of contact of the circle AD, and the wheel or generating circle, and therefore BV = AO. Alſo ſince WT is $\perp$ to TV, therefore V is the point of contact of the generating circle (whoſe diameter is VW) and OS, whence VW = OR. The figures are ſimilar, becauſe their axes are as the radii of the ſpheres.

[Pr. 52. hence ſince in unequal—] ſuppoſe TR
30. to be a ſmaller oſcillation, to find the velocity in I; take *st* : SR : : TI : TR; then velocity at R in the arc TR : velocity at R in SR : : TR : SR; and velocity at I in TR : velocity at R in TR : : (Prop. 51.) velocity at *t* in SR : velocity at R in SR : :) $\sqrt{SR^2 - tR^2}$: SR. And *ex equo* velocity at I in TR : velocity at R in SR : : TR$\sqrt{SR^2 - tR^2}$: SR^2 : : $\frac{TR}{SR}\sqrt{SR^2 - tR^2}$: SR : : (becauſe SR : TR : : *t*R : IR as) $\sqrt{TR^2 - IR^2}$: SR.

[ib. There are obtained from the times given] Fig. for in two unequal arches, two correſponding parts are deſcribed in equal times (by Prop. 51.) therefore the velocities in theſe points will be as the entire arcs; and therefore both the velocities and arches will be known.

[ib. And if the abſolute force of any globe] that is, the force at a given diſtance be called V; then the force at the diſtance CO is CO × V. But in a given time, HY is as the force; whence CO × V is as HY, and therefore HY is deſcribed in a given time.

[ib. cor. 1. For this time] for then AR = AC, and V is given; therefore, this time : time of $\frac{1}{2}$ oſcillation $: : \sqrt{\frac{AC}{AC}} : \sqrt{\frac{AR}{AC}} : : 1 : \sqrt{\frac{AR}{AC}}$.

[ib. cor. 2. But in that caſe] for (in fig. 2. pl. 19.) by cor. 1, 2, pr. 49. $AS : Bv : : PS : PV : : 2CE : CB$ (in this caſe as) : : 2 : 1 : : (and by diviſion) $AP : BV - PV$; whence $AP = 2BV - 2PV$; but verſed ſine of $\frac{1}{2}PB = \frac{BV - PV}{2}$, for coſine $= \frac{1}{2}PV$; and $\frac{1}{2}VB - \frac{1}{2}VP =$ verſed ſine.

[ib. as M. Huygens] all this is demonſtrated in Keil's Philoſophy. See my large book of Mechanics, Prop. 40. cor. 4.

[Pr. 56, let the projection] T is projected into P and t into p.

[ib. as alſo its poſition] The greater axis $2Tt$ is perpendicular to PO.

[ib. and ſince the area POp] for let $\frac{\text{area } OPp}{OP} = sp$, 31. which is given ($sp$ being ⊥ to PO); and $PB^2 : Pr^2 : : PB^2 - SP^2 : PS^2$; whence Ps is given, and thence SO, the angle POp, Op and Pp, the point p, and angle OPp.

SECT. XI.

[Pr. 57, and about each other] conſidering either body as at reſt.

Fig. [Pr. 58, by the ſame forces there may be deſcribed, &c.] that is, by making the velocity to the former velocity as ($\sqrt{sp}$ to $\sqrt{\mathrm{CP}}$, that is as) $\sqrt{\mathrm{P}+\mathrm{S}}$ to $\sqrt{\mathrm{S}}$, as appears by the Prop.

[Pr. 60, in a ratio ſeſquiplicate] it ſhould be ſubſeſquiplicate.

[ib. of the other ellipſis,] As $\mathrm{S}^{\frac{1}{3}} : \overline{\mathrm{S}+\mathrm{P}}^{\frac{1}{3}}$. Let $\mathrm{S}^{\frac{1}{3}} : a : e : \overline{\mathrm{S}+\mathrm{P}}^{\frac{1}{3}}$ be ∺. Then $\mathrm{S}^{\frac{1}{3}} : \overline{\mathrm{S}+\mathrm{P}}^{\frac{1}{3}} : :$ ($\mathrm{S} : a^3 : :$) $e^3 : \overline{\mathrm{S}+\mathrm{P}}$. But (ſince $\overline{\mathrm{S}+\mathrm{P}} : e^3 : : a^3 : s$) e^3 is the firſt of two mean proportionals between $\mathrm{S}+\mathrm{P}$ and S.

[Pr. 61.] Note, if a body were placed in the center of gravity, and whoſe force would be ſufficient to cauſe one of the bodies to revolve around it, and to deſcribe the ſame figure; yet it would not cauſe the other body (if it were unequal) to deſcribe its figure, except in that law of centripetal force, which is as the diſtance where the periodic times are all equal. For, to preſerve the ſame motion as before, either body muſt be attracted to the body in the center of gravity, with a force which is as the other body, or as its own diſtance, which is the caſe of one body attracting another; and holds only (in the caſe of their being attracted by a third body in the center of gravity) in the law of centripetal force before-mentioned. And therefore in

[Pr. 62.] The foregoing note is to be obſerved.

[Pr. 64. This would be the caſe] For this only cauſes the bodies T, L, to revolve more ſwiftly round their center of gravity D, but affects not the other bodies.

[ib. with equal periodical times.] By the foregoing (in the three bodies S, T, L) the body S deſcribes an ellipſis round C; and, by conſidering the center of gravity of S, T, as deſcribing an ellipſis (as before of T, L,) it will appear the ſame way that the body L deſcribes an ellipſis round C; and as the center of gravity of S, L, deſcribes an ellipſis, ſo T deſcribes an ellipſis alſo, round C. Fig. Further, from what went before, the centripetal force of S towards C, is as $\overline{T+L} \times \overline{SC+CD}$, or $\overline{T+L+S} \times SC$. Alſo the centripetal force of T towards D, is as $L \times \overline{TD+DL} + S \times TD$ (which latter part ariſes from the reſolution of the force ST into SD, DT, and DT acts towards D), or as $\overline{T+L+S} \times TD$. Now by cor. 2, 8. pr. 4. The periodic times are in the ſubduplicate ratio of the radii directly, and the ſubduplicate ratio of the forces inverſly: therefore the periodic time of S, round C : periodic time of T, round D : : $\sqrt{\frac{SC}{SC\times\overline{T+L+S}}} : \sqrt{\frac{TD}{TD\times\overline{T+L+S}}}$; and are therefore equal; ſo that after one revolution, the bodies all return to their firſt places.

After the ſame manner the point C, and a fourth body V, as alſo the bodies S, T, L, will deſcribe ellipſis round their common center of gravity B, for any one, and the center of gravity of the other three will deſcribe ellipſes; and the caſe is the ſame if there were more bodies. Alſo, as before, and by a like reſolution of forces, ſince the forces VS, VT, VL, are reſolved into VC, CS; VC, CT; VC, CL; the former acting in the direction of CV towards V, the latter to C their center of gravity; and are as the diſtances of the bodies therefrom, as before; they will therefore ſtill move round their center of gravity C as before, but ſwifter, altho' all the four revolve round B. Alſo periodic time of V round B : periodic time of S round C : : $\sqrt{\frac{BV}{\overline{S+T+L}\times\overline{CB+BV}}} : \sqrt{\frac{SC}{\overline{T+L\times S}\times SC+V\times SC}}$: : (that is, as) $\sqrt{\frac{BV}{\overline{T+L+S+V}\times BV}} : \sqrt{\frac{SC}{\overline{T+L+S+V}\times SC}}$; and are therefore equal;

and

Fig. and if more bodies were added, all the bodies would perform their revolutions in equal periodic times. *Q. E. D.*

[Pr. 66. cor. 2. Such is the force NM.] This force always acts from P in a direction parallel to TS, and from M towards T.

[ib. cor. 6. the periodic time will be increaſed,] for then the periodic time is $\propto \frac{\text{Rad}^{\frac{3}{2}}}{\sqrt{\text{force}}}$, which is a greater ratio than $\text{Rad}^{\frac{3}{2}}$, becauſe the force is diminiſhed.

[ib. cor. 7. the upper apſis to go backwards] for $\frac{n^2}{m^2} - 3 > -2$. And $\frac{n^2}{m^2} > 1$; therefore n is $>$ than m. Alſo put $D =$ diſtance ($r =$ a given quantity,) and the force is $\Big(\frac{D^3+r^3}{D^3}$, or rather) as $\frac{1}{D^2+\frac{1}{D}}$, and when the diſtance increaſes, the decreaſe of the force is $D^2 + \frac{1}{D}$, which is leſs than $D^2 + \frac{D^3}{D}$, which is the decreaſe according to the duplicate ratio. Alſo at the conjunction, &c. the force of P towards T is, the centripetal force of T + LM — TM, or the centripetal force of T — KL. Alſo the decreaſe of the force is as $d^2 - \frac{1}{d}$, which is greater than $d^2 - \frac{d^3}{d}$, or the decreaſe according to the duplicate ratio. And $\frac{n^2}{m^2} - 3 < -2$; and $n < m$.

[ib. cor. 7. The truth of this] for then the further P recedes from T the more it is attracted towards the bodies S, S, and therefore leſs towards T, than it would otherwiſe be.

[ib. cor. 8. when the apſides are in the ſyzyges] Fig. for then NM is greater, and LM leſs, than before.

[ib. cor. 9. Now therefore —] For the ratio of KL to LM is leaſt when the apſides are in the quadratures, and greateſt in the ſyzyges.

[ib. For the forces LM] as is ſhewn in cor. 7.

[ib. cor. 10, from the ſyzyges to the quadratures] for after the ſyzyges, the body P by the action of S, is made to move in lines which ſucceſſively cut the plane TPS at greater and greater angles. And for the ſame reaſon at the 90° from the octants (between C and A), the body is drawn into lines (or little planes) which cut TPS at greater and greater angles, which before 90° cut it at leſs and leſs.

[ib. and by a like reaſoning] for the inclination 32. is increaſed from *u* to D, diminiſhed from D to S, increaſed from S to C, diminiſhed from C to *u*; as is plain by ſuppoſing T and any point *p* wherein the body is, to be joined; and the plane *p*T*t* to revolve round *p*T, till it paſs thro' the new place of the body *p*, (which it acquires either above or below the plane *p*T*t*,) by the (action of S upon it, or the) force NM; and then it will appear, how that new acquired plane cuts the plane SCT, whether in a greater or leſſer angle.

[ib. cor. 11.—from the former plane CD] Since the inclination of the orbit is diminiſhed from C to A, the interſection with TSE will move from C to B, and from D to A; and ſeeing that inclination is increaſed from A to D, the interſection (or nodes) will ſtill move towards B and towards A.

[ib. being always either retrograde] that is, conſidering a whole revolution; for in ſome points they go forward.

[ib. cor. 13. And ſince the cauſes and proportions] to one another, and to the force of S, &c.

Fig. [ib. cor. 14. But ſince the forces] For SK : LM : : accelerating force of T towards S : perturbating force of S. And therefore the perturbating force of S $= \frac{LM \times \text{accelerating force of T towards S}}{SK}$.

But ſince SK = ST nearly, and LM (in its mean quantity) = PT. And the accelerating force of T towards S, is as the body S directly, and ST^2 reciprocally; therefore the perturbating forces LM, NM $\propto \frac{PT \times \text{body S}}{ST^3}$, that is, $\propto \frac{PT \times \text{bodies}}{\square \text{ periodic time of T round S}}$. Or $\propto PT \times$ denſity of S $\times$ cub. apparent diameter of S. (for the apparent diameter is as the real diameter directly, and the diſtance reciprocally)

[ib. cor. 16, ſquare of the periodical time of the body P conjunctly.] This holds as true in the ſynodic time of the body P, (ſince the action of the forces that cauſe theſe errors begin and end at the quadratures, which comes to the ſame, as if they begun and ended at the ſyzyges) ſee what follows.

[ib. and hence the angular errors] theſe in one revolution are as the linear errors directly, and the radius or diſtance reciprocally, that is, as the forces and ſquare of the time of revolution of P, directly, and the diſtance TP reciprocally; that is (becauſe the force $\propto$ TP) as ſquare of the time of revolution.

[ib. Let theſe ratio's] The angular errors of P obſerved from T are as $\frac{\text{forces} \times \text{time}^2}{TP}$, that is (by cor. 14.) as $\frac{PT \times \text{body S} \times \text{time}^2}{PT \times \square \text{ T's periodic time}}$, that is, in the time of one revolution of P (and if S be given), as $\frac{\square \text{ time of P's revolution}}{\square \text{ periodic time of T}}$.

[ib. both theſe motions will be as the periodical time of the body P, directly,] for the angular motion or velocity, or the mean angular errors, are as the ſum of all the angular errors in any time directly, and the time reciprocally; that is, (in the periodic time) as Fig. 32.

$$\frac{\text{body } S \times \square \text{ time of P's revolution}}{\text{time of P's revolution} \times \square \text{ periodic time of T}},$$

that is, as body S × time of P's revolution directly; and the ſquare of the periodic time of T, reciprocally.

[Cor. 20. And thence the greateſt height of the water] This is explained by Worſter in his (Experimental) Philoſophy, page 72. Alſo ſee my Geography, Prop. 12. Sect. I.

SECT. XII.

[Pr. 79. As the lineola D*d*,] when the radius of the ſphere is given; and when the radius is not given, as D*d* and the radius conjunctly; therefore AS, and alſo PS or PE, may be of any magnitude.

[ib. the ſum of all the rectangles PD × D*d*] that is, as $\frac{PF + PD}{2} \times DF$, or $\frac{PF + PD}{2} \times \overline{PF - PD}$, or $\frac{PF^2 - PD^2}{2}$.

[Pr. 80. The whole force of the ſphere will be as the whole area ANB.] Alſo it is manifeſt, that the attraction or force of any part of the ſphere FEB, is as the correſpondent part of the area DNB.

[Pr. 81. Ex. 1.] The area SL into AB = SL × AB, becauſe both SL and AB are given. And the area LD into AB is a trapezoid, whoſe baſe is AB, and the two parallel ſides LA and LB; for as the point D removes nearer A, the ordinate LD

Fig. 32. decreaſes (from B where it is LB) to A, (where it is LA); and that area is $\frac{\overline{LA}+\overline{LB}}{2}\times AB$, or $\frac{\overline{LA+LB}}{2}\times\overline{LB-LA}$, that is, $\frac{LB^2 - LA^2}{2}$; or it is $\frac{2LA+AB}{2}\times AB$, that is LS × AB. And the ordinate $\frac{ALB}{LD}$, is the ordinate of an hyperbola between the aſſymptotes, whoſe center is L, and one aſſymptote LB; for ALB is a given rectangle, and the nature of the hyperbola is that $\frac{ALB}{LD}$ (LD the part of the aſſymptote being taken as the axis) is as the ordinate erected on D; whence, the reaſon of the conſtruction will be evident; for *a*AB*b* is the aforeſaid trapezoid, by conſtruction; alſo in the hyperbola *ab*, the rectangle LB × B*b* is given, but that (by conſtruction) is = the rectangle ALB; when the hyperbolic area *a*AB*b* is that deſcribed by the ordinate $\frac{ALB}{LD}$. And the difference of that and the trapezoid (which is the difference of the two firſt areas) is the area *aba* required.

[ib. Ex. 2. the third $\frac{ALB\times SI}{2LD^2}$) this is eaſily calculated by fluxions, LD is a flowing quantity; the fluxion of the area is $\frac{ALB\times SI}{2}\times LD^{-2}\times \dot{LD}$. And the whole area is $\frac{ALB\times SI}{-2}\times LD^{-1}$, that is $\frac{ALB\times SI}{-2LD}$, or (when LD = LB) $\frac{ALB\times SI}{-2LB}$: from this take the area (when LD = LA, *viz.*)

ALB

$\frac{ALB \times SI}{-2LA}$, and the difference is the area ſought Fig. $\frac{ALB \times SI}{2LA} - \frac{ALB \times SI}{2LB}$.

[ib. Ex. 3.] The fluxion of the firſt area is $\frac{SI^2 \times SL}{\sqrt{2SI}} \times LD^{-\frac{2}{3}} \times L\dot{D}$, and the whole area is $\frac{2SI^2 \times SL \times LD^{-\frac{1}{2}}}{-\sqrt{2SI}}$, or $\frac{2SI^2 \times SL}{-\sqrt{2SI} \times \sqrt{LB}}$, and that part of the area on AB is $\frac{2SI^2 \times SL}{\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA}} - \frac{1}{\sqrt{LB}}}$. The fluxion of the ſecond area is $\frac{SI^2}{2\sqrt{2SI}} \times LD^{-\frac{1}{2}} \times L\dot{D}$; and the whole area $\frac{SI^2}{\sqrt{2SI}} \times LD^{\frac{1}{2}}$, and the area on AB is $\frac{SI^2}{\sqrt{2SI}} \times \overline{\sqrt{LB} - \sqrt{LA}}$. Alſo the fluxion of the 3d area is $\frac{SI^2 \times ALB}{2\sqrt{2SI}} \times LD^{-\frac{5}{2}} \times L\dot{D}$; and the whole area $\frac{SI^2 \times ALB}{-3\sqrt{2SI}} \times LD^{-\frac{3}{2}}$; and the area on AB is $\frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA^3}} - \frac{1}{\sqrt{LB^3}}}$.

[ib. And thoſe after due reduction] LA, LI, LB are ∺; for SP $= \frac{SA^2}{SI}$. PI $= \frac{SA^2}{SI} - SI$. LI $= \frac{SA^2 - SI^2}{2SI}$. And LA $= PS - AS - LI = \frac{SA^2 + SI^2}{2SI} - SA$. And LB $= \frac{SA^2 + SI^2}{2SI} + SA$. But $\frac{SA^2 + SI^2}{2SI} - SA : \frac{SA^2 - SI^2}{2SI} :: \frac{SA^2 + SI^2}{2SI} +$ SA, Fig. SA, are ∺, as will appear by multiplying them. Hence $\surd LB - \surd LA = \surd 2SI$. For $2\surd\overline{LB \times LA} = 2SI = 2LS - 2SI = LB + LA - 2SI$. And $LB + LA - 2\surd\overline{LB \times LA} = 2SI$. And (by Evolution) $\surd LB - \surd LA = \surd 2SI$. Therefore the firſt area $\left(= \frac{2SI^2 \times SL}{\surd 2SI} \times \frac{\surd LB - \surd LA}{\surd\overline{LB \times LA}}\right) = \frac{2SI^2 \times SL}{LI}$. And the ſecond area $= SI^2$. And $\surd LB^3 - \surd LA^3$ $(= \overline{LB + LA} \times \overline{\surd LB - \surd LA} + \surd LB \times \surd ALB - \surd AL \times \surd ALB = 2LS \times \surd 2SI + \surd ALB \times \surd 2SI = \overline{2LS + LI} \times \surd 2SI) = \overline{3LI + 2SI} \times \surd 2SI$. Whence the third area $= \frac{SI^2 \times ALB}{3\surd 2SI} \times \frac{\surd LB^3 - \surd LA^3}{\surd ALB^3} = \frac{SI^2 \times \overline{3CI + 2SI}}{3\surd ALB = 3LI} = SI^2 + \frac{2SI^3}{3LI}$.

[Pr. 83. let us ſuppoſe] it ſhould be, let that ſuperficies be not a purely, &c. as it is in the original.

SECT. XIII.

[Pr. 90. cor. 1, 2.] for the fluxion of the area is as $D^{-n}\dot{D}$, and the area as $\frac{D^{1-n}}{1-n}$ or as $\frac{1}{D^{n-1}}$; and the area ALIH, as $\frac{1}{PA^{n-1}} - \frac{1}{PH^{n-1}}$.

[Pr. 91. cor. 1. And the other part $\frac{PF}{PR}$] for the fluxion of that area is $= \frac{PF}{PR} \times P\dot{F} = \frac{PF}{\surd RF^2 + PF^2} \times P\dot{F} = \overline{RF^2 - PF^2}^{-\frac{1}{2}} \times PF \times P\dot{F}$. And the area $= \overline{RF^2 + PF^2}^{\frac{1}{2}} = PR$. And (the area on

PB is PE—AD; and on PA, PD—AD. And Fig. their difference) PE—PD, is the area on AB.

[Pr. 91. cor. 2.] This is calculated in the Appendix, by the help of the Quadratures of Curves, which ſee in Harris's Lex. vol 2.

And the curve KRM is a conic ſection; for the indetermined quantities riſe only to two dimenſions. Let SA $= r$. SC $= c$. BE $= u$. PB $= b$.

$$ER^2 = PD^2 = PE^2 + ED^2 = \overline{b-u}^2 + \frac{cc}{rr} \times \overline{2ru - uu} = bb + \overline{\frac{2cc}{r} - 2b} \times u + \overline{1 - \frac{cc}{rr}} \times uu.$$

[Sch. Pr. 93, and I ſuppoſe the force proportional.] 33. For conſider A, B, as flowing quantities, and let $\dot{A}$ be given $= o$. Now ſince $A^{\frac{m}{n}} = B$; therefore when A becomes $A + \dot{A} = A + o$, B becomes $B + \dot{B}$, and then $B + \dot{B} = \overline{A+o}^{\frac{m}{n}} = A^{\frac{m}{n}} + \frac{m}{n} oA^{\frac{m-n}{n}} + \frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}} +$ &c. Now if any quantity $\overline{A+o}^{\frac{m}{n}}$ (where o is infinitely ſmall) be involved; the firſt, ſecond, third, &c. term, will be reſpectively as the flowing quantity ($A^{\frac{m}{n}}$), the firſt, ſecond, &c. fluxion of that quantity, as is demonſtrated in the quadrature of curves, Sch. to Pr. 11. (for which ſee Harris's Lexicon.) Therefore $\frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$ is as the ſecond fluxion of $A^{\frac{m}{n}}$. Now if a (centrifugal) force act from the line CD in the direction B, and a body move in the curve CZ, the fluxion of A will continue the ſame always, and the force acting upon the body will be every where as the ſecond fluxion of B, or of

Fig. of $A^{\frac{m}{n}}$ (its equal), and therefore as $\frac{m^2 - mn}{2n^2}$ $ooA^{\frac{m-2n}{n}}$, or (becauſe o is given,) as $\frac{m^2-mn}{n2} A^{\frac{m-2n}{n}}$, or as $\frac{m^2-mn}{n^2} B^{\frac{m-2n}{m}}$. Or $\frac{mn-m^2}{n^2} B^{\frac{m-2n}{m}}$ is as the centripetal force. This may be eaſilier ſhewn by fluxions only, by finding the 2d fluxion of (B or) $A^{\frac{m}{n}}$.

SECT. XIV.

34. [Pr. 94. equal to the ſquare of HM] for imagine
35. a diameter drawn through H, and an ordinate to it through I, the abſciſſa will be = MI, and the ordinate = MH. Alſo compleating the figure, the tangent IP cuts from the diameter H*a*, H*p* = H*u* = MI; therefore the triangles HLP, MLI are equal and ſimilar, and HL = LM. And the latus rectum to H*a* is always the ſame, whatever the angle of incidence be, if the velocity be given; for the line H*d*, that the body will deſcribe in a given time, is given; and the line *dg*, that the body approaches the plane in a given time is given; and therefore $\frac{Hd^2}{Hz}$, or $\frac{\overline{zg}^2}{zH}$, or the latus rectum, is given.

[Pr. 96. Sch. of the ſecants] of the angles between the line of incidence and the plane.

[Pr. 97, cor. 2.] The curve lines CP, CQ (being every where perpendicular to AP, DK,) are compoſed of arches of circles; therefore PD is the increment of AP or AC, and QD the decrement of QK or CK; and therefore theſe increments are as the ſines of incidence and emergence. And *è contra*, if PD, QD are as the ſines of incidence and emergence, a body moving in the line PD, ſhall emerge in the line QDK, by this Prop.

[Pr. 98. and therefore QS to be always equal to CE;] For ſince *qqqq* and *ssss* are always perpendicular to *qs*, therefore ſuppoſing QS and 1*qs* to interſect in K; then KQ — KS = K.1*q* — KS, that is *qs* = 1*q.s*. Alſo let 1*q*.K and 2*q.s* interſect in T, then 1*q.s* = 1*q*.T — ST = 2T — *s*T = 2*q.s*. Alſo 2*q.s* = 2*q*.V — SV = 3*q*.V — SV = 3*q*.S; and ſo on, 'till at laſt *qs*, coinciding with CE, will be equal thereto. Therefore QS is every where equal to CE. Fig. 36.

[ib. Sch] See the Author's Optics, where all theſe things are ſhewn.

BOOK

BOOK II.

SECT. I.

[PR. 2. cor. For if that area] by Marq. Hoſpital's Conic Sect. art. 219. And my Conic Sect. Prop. 86, Hyperbola, cor. 6.

[ib. of the right line AC] by the converſe of Lem. 1; but the contrary does not hold, that is, if any area ABGD be taken for the time, that DC ſhall repreſent the velocity, or AD the ſpace.

[Pr. 3. And the reſiſtance] The force of gravity may be compared with the reſiſtance of the medium; for they may both be conſidered as uniform preſſure.

[ib. or AC to $\frac{1}{2}$AK] For ſuppoſe a tangent to be drawn to the point B, then Kq : (ſubtangent =) AC : : kq : Bk or AK. And in general in any area, as M$st$N; it will be, AC : $\frac{1}{2}$MN + NC : : (by the nature of the hyperbola) $\frac{Ms + Nt}{2}$: AB or Mm. And by diviſion AC : AM + $\frac{1}{2}$MN : : $\frac{Ms + Nt}{2}$: $\frac{ms + nt}{2}$: : area $s$MN$t$ : area $smnt$. But AG : AM + $\frac{1}{2}$ MN : : force of gravity : reſiſtance in the middle of the fourth time. Therefore, $s$MN$t$ : $smnt$: : force of gravity : reſiſtance in the midſt of the fourth time, and ſo of the reſt.

37. And the like demonſtration holds in aſcending motion; for let the rectangle ABDG be divided into innumerable rectangles Dk, Kl, Lm, Mn, &c. which ſhall be as the decrements of the velocities produced in ſo many equal times; then will AE, Ak, Al, Am, &c. be as the whole velocities, and therefore as the reſiſtances of the medium in the

beginning of each of the equal times. Let AC : Fig. 37. AK : : force of gravity : reſiſtance in the beginning of the ſecond time; then to the force of gravity add the reſiſtances, and DEHC, K*k*HC, L*l*HC, M*m* HC, &c. will be as the abſolute forces, or ∝ decrements of the velocities, ∝ D*k*, K*l*, L*m*, M*n*, &c. and therefore ∺ ; whence DG*q*K = K*qr*L = L*rs*M = M*st*N, &c. will be ∝ the equal times or forces. But AC : MC : : M*s* : AB. And by diviſion AC : AM : : M*s* : *ms* : : area *s*MN*t* : area *smnt* : : (and therefore as) force of gravity : reſiſtance in the fourth time; and ſo of the other areas. Therefore ſince DG*q*K, K*qr*L, L*rs*M, M*st*N, are ∝ the gravitating forces, the areas GE*kq*, *qklr*, *rlms*, *msnt*, ∝ reſiſtances in each time, ∝ velocities ∝ ſpaces deſcribed. And by compoſition, in the times DG*r*L, DGBA, the ſpaces deſcribed will be as GE*lr*, GEB. *Q. E. D.*

[ib. cor. 3.] For the diff. ſpaces ∝ velocities (that is by cor. 2.) ∝ CA, CK, CL (∺), or ∝ AK, KL, LM (∺) by lem. 1. (fig. 3.)

[ib. let that alſo be diſtinguiſh'd] This is true in this law of reſiſtance; becauſe the motion loſt is ∝ velocity ∝ remaining velocity ∝ ſpaces deſcribed.

[Pr. 4. which is the locus of the point *r*. *Q. E.D.*] all the reſt is plain by Prop. 2. and 3, and cor. Pr. 2.

[ib. cor. 1. that is, if the parallelogram] For DA : CP : : DR : RX $= \frac{DR \times CP}{DA}$. But DAB $= \overline{DC - AC} \times AB =$ (by the nature of the hyperbola) $\overline{AB - AQ} \times DC = QB \times DC =$ (by conſtruction) $N \times CP$. And $DA = \frac{N \times CP}{AB}$ Wherefore $RX = \frac{DR \times CP \times AB}{N \times CP} = \frac{DR \times AB}{N}$.

[ib. cor. 4. D*ra*F is alſo given.] by this prop. [ib. cor. 6.] for then $2DP \propto \frac{\text{lat. rect.}}{\text{reſiſtance}} \propto \frac{\text{lat.}}{\text{velocity}} \propto \frac{\text{velocity}^2}{\text{velocity}} \propto \text{velocity}$.

[ib. cor. 7. the ratio $\frac{Ff}{DF}$] For when this ratio is the ſame with the other, the curves deſcribed (and ſuppoſed to be deſcribed) in theſe two caſes, will be ſimilar.

SECT II.

Prop. 5. had been better expreſſed thus, (tho' the tranſlation agrees with the original.)

If a body is reſiſted in the duplicate ratio of its velocity, and moves by its innate force only, thro' a ſimilar medium; and the ſpaces be taken equal: I ſay, that the times of their deſcription are in a geometrical progreſſion increaſing; and that the velocities at the beginning of each of the times are in the ſame geometrical progreſſion (decreaſing or) inverſely. For it is plain, that the times may be taken in ſuch a gometrical progreſſion, as that the ſpaces cannot be equal, nor the velocities in the ſame inverſe progreſſion. But if the ſpaces are equal, the times and velocities will be in a geometrical progreſſion, inverſe to one another. The demonſtration is the ſame however the propoſition be expreſſed. For it being proved that the lines AB, K*k*, L*l*, &c. being ſquared, the ſquares are as the differences of the lines; and alſo that the ſquares of the velocities are as the differences of the velocities. Therefore if AB and K*k* be taken as the velocities in the beginning of the times AK, KL; and an hyperbola be drawn thro' the points B, *k*, to the aſſymptote CD; (whoſe center let be C; all the other velocities will be as the lines L*l*, M*m*, &c.

becauſe the progreſſion, of the lines as well as the velocities, continues the ſame all along. Whence the ſpaces (in theſe equal times) will be as the areas A*k*, K*l*, L*m*, &c. and in the time AM, the ſpace will be as AM*m*B. Now conceive the area AM*m*B to be divided into the equal areas A*k*, K*l* L*m*, &c. Then will CA, CK, CL, &c. be ∺' increaſing; and the parts (by Lem. 1.) AK, KL, LM, &c. which are as the times, will be in the ſame progreſſion, increaſing. Alſo the velocities AB, K*k*, L*l*, &c. (which are reciprocally as CA, CK, CL, &c.) will be in the ſame progreſſion inverſely. *Q. E. D.* Fig.

[Pr. 7, and times conjunctly] very ſmall particles of time.

[ib. they will always deſcribe ſpaces,] for the parts of ſpace deſcribed in thoſe ſeveral particles of time, are as the reſpective velocities × thoſe ſeveral parts of time; that is, as the firſt velocity × each part of time. And by compoſition, the whole ſpaces are as the firſt velocities × whole times. *Q. E. D.*

[Prop. 7, cor. 3.] dele. Let thoſe diameters applied to that power.

[Pr. 8.] in fig. 1, Pl. 2, *k* ought to be beyond *l* in reſpect of A, &c. becauſe the reſiſtance decreaſes. But the demonſtration is the ſame any way. But this figure is not in the 1ſt or 2d Ed. of the original. In Prop. 8 and 9, AC repreſents the relative gravity, or its weight in the fluid.

[Pr. 9. cor. 7.] For the greateſt velocity is given (by Pr. 8. cor. 2, 3.) And thence the time of acquiring that velocity in free ſpace. And ABNK (AB*nk*) being given there is given AK, and AP (A*p*) and ATD (A*t*D). Then ADC : ADT (AD*t*) : : time of acquiring that greateſt velocity in free ſpace : time ſought.

[Pr. 10. equal among themſelves] and very ſmall.

Fig. [ib. or $\frac{MI \times NI}{HI}$] as appears by ſimilar △'s, produced by letting fall a ⊥ from N on HI.

If the velocity in GH be greater than the velocity in HI, the decrement is $\frac{GH}{T} - \frac{HI}{t}$, and ariſes from the reſiſtance and gravity together (becauſe gravity draws the body from the tangent into the arch HI); and if gravity act not, that decrement would be greater (for gravity accelerates the motion), by $\frac{2MI \times NI}{t \times HI}$; therefore the decrement, by the reſiſtance alone is $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times NI}$.

[ib. will be NI;] for Q being a given (ratio or) quantity; Qo will always repreſent MN, whatever the magnitude of o be.

And the ordinate DI = CH — MI. Alſo the value of MI in the ordinate EK is (becauſe o becomes $2.o$; and ſubſtituting $2 \times o$, for o,) $2Qo + 4Ro^2 + 8Ro^3 +$ &c. and therefore $EK = CH - 2Qo - 4Ro^2 - 8So^3 -$ &c. Alſo in the ordinate BG, when o becomes $-o$, the value of MI (by ſubſtituting $-o$ for o) will be $-Qo + Ro^2 - So^3 +$ &c. And BG = CH — MI (as before,) $= P + Qo - Ro^2 + So^3 -$ &c.

[ib. or $\frac{R + \frac{3}{2}So}{R}$] for (becauſe o is infinitely ſmall) R, $R + \frac{3}{2}So$, $R + 3So$, are both arithmetical and geometrical proportionals.

[ib. by ſubſtituting the values —] becomes

$$\overline{1 + \frac{3So}{2R} \times o\sqrt{1+QQ} - \frac{QRco}{\sqrt{1+QQ}} - o\sqrt{1+QQ}}$$

—

$-\frac{QRoo}{\sqrt{1+QQ}} + \frac{\overline{2Qo+2Ro^2}\times\overline{Ro^2+So^3}}{o\sqrt{1+QQ}+\frac{QRoo}{\sqrt{1+QQ}}} =$ (neglecting the ſuperfluous powers of o, or the quantities infinitely leſs than the other,) $o\sqrt{1+QQ} - \frac{QRoo}{\sqrt{1+QQ}} + \frac{3So^2}{2R}\sqrt{1+QQ} - o\sqrt{1+QQ} - \frac{QRo^2}{\sqrt{1+QQ}} + \frac{2QRoo}{\sqrt{1+QQ}} = \frac{3Soo}{2R}\sqrt{1+QQ}.$

[ib. latus rectum $\frac{HN^2}{NI}$] or $\frac{HI^2}{NI} =$ (neglecting the ſuperfluous powers of o) $\frac{oo\times\overline{1+QQ}}{Roo} = \frac{1+QQ}{R}.$

[ib. p. 33, that a body by aſcending from P] or any other point of the quadrant PF, in the direction of the arch of that quadrant.

[ib. Ex. 3. the ſecond term $\frac{m}{n}o - \frac{bb}{aa}o$] or rather $\frac{bb}{aa}o - \frac{m}{n}o = Qo$; but the ſquare is the ſame either way.

[ib. Ex. 4. for Qo] $\frac{nbb}{A^{n+1}}o - \frac{d}{e}o = Qo.$

[ib. Sch. And therefore if a curve] for the quantity $\frac{S}{R^{\frac{4-n}{2}}} \div \frac{\overline{HT}|^{n-1}}{AC}$ is as the denſity, and if that be given the denſity is given.

[ib. than theſe hyperbola's here deſcribed.] For let the 2 mediums be of the ſame denſity in the vertex of the curves; then if the bodies be projected from A; the uniform medium being more denſe, and the other leſs denſe at A, the body moving in the uniform medium will be more impeded, and

Fig. and conſequently will deſcend the more, than that deſcribing the hyperbola, which is leſs reſiſted, and which therefore moves nearer a ſtreight line. Therefore the body moving in the uniform medium is more diſtant from the aſſymptotes in the vertex of the figure than the other body. But if the denſities of the mediums were ſuppoſed equal in A; then (by the ſame way of reaſoning), the body in the uniform medium would be nearer the aſſymptotes in the vertex. And generally ſpeaking, theſe curves cannot therefore differ very far.

[ib. will touch the hyperbola in G,] for ſuppoſe the tangent TG to interſect MX. Then (by fluxions) the diſtance between that point of interſection and V, is $= \frac{VX}{n}$. Then by ſim. Δ's, $\frac{VX}{n} : VG :: VX : TX - VG = nVG$. But $VY = nVG$. Therefore $TX = GY$.

[ib. and the velocity] for the velocity (when the gravity is given) is as the ordinate of the parabola, that is (the abſciſſa being given, by the gravity) as the $\sqrt{\text{latus rectum}}$.

[ib. Rule 1.] For the velocities are as the √'s of the latus rectum of two parabola's, that is, as $\sqrt{\frac{2XY^2}{nn+n \times VG}}$, that is, as $\sqrt{\frac{2AH^2}{AI}}$ to $\sqrt{\frac{2Ab^2}{Ai}}$ which (by ſuppoſition) are in a ratio of equality, and therefore $\frac{AH}{AI} = \frac{Ab}{Ai}$. Alſo ſince the denſities are equal; $\frac{n+2}{3XY}$ will be given, that is $\frac{n+2}{3AH} = \frac{n+2}{3Ab}$. Whence $AH = Ab$. Whence AH, AI, remain the ſame; and HX, which is compoſed of AI and the ſubtangent to the axis XH, is the ſame alſo.

[Rule 2.] For velocity $\propto \sqrt{\frac{AH^2}{AI}} \propto$ (becauſe AH is given) $\sqrt{\frac{1}{AI}}$. Fig.

[Rule 3. and therefore AH] for let b be the ratio, then $\frac{Ab}{Ai} = b\frac{AH}{AI}$. And $\frac{AH^2}{AI} = \frac{Ab^2}{Ai} = b\frac{AH}{AI} \times Ab$. Whence $Ab = \frac{AH}{b}$. Alſo $\frac{Ab}{Ai} = \frac{AH}{bAi} = b \times \frac{AH}{AI}$. Whence $Ai = \frac{AI}{bb}$.

[Rule 4. a little greater] ſince AH decreaſes faſter than GT, therefore the ſum of all or every AH + GT is leſſer than the firſt AH + GT × number of them. And therefore the ſum of all the denſities (being reciprocal thereto) is greater than the ſum of as many denſities in A and G. But the mean denſity, is = ſum of all the denſities ÷ by the number of them, and therefore is ſomething greater than the ½ ſum of the denſities in A and G. But half ſum denſities : denſity in A : : $\frac{AH + GT}{2}$; GT. Therefore mean denſity : denſity in A : : is in oratio a little greater than $\frac{1}{2} \times \overline{AH + GT} : GT$.

[ib. Rule 5.] for HX = AI + ſubtangent = AI + nAI. XN being the axis.

[ib. Rule 6. by how much] for the variation of curvature in one of theſe hyperbola's, where n is great, is leaſt in the part AG, and ſo it is in the curve of projection, where the velocity is greateſt; ſo they agree in that part. But they differ more in the part GK, where the projectile approaches to a uniform motion.

[ib. Rule 7.] AH to 2AI, ought to be 3AH to 4AI. (But the original is AH to 2AI.)

Fig. [ib. Rule 8. whoſe conjugate] hyperbola ſhall paſs thro' the point C.

[ib. what has been ſaid] for by ſuppoſing the index n to be negative, VG (which before was as VX^{-n}) will then be as VX^{n}, and the curve AG will be a parabola, and what was demonſtrated generally of the hyperbola, will hold true (for any index, and therefore) for the parabola; and all things will follow as before ſubſtituting $-n$ for n.

38. But the computation may be made for the parabola, as for the other; thus, let $VG = bbXV^{n} = XC^{n}$. Then the ſubtangent $ZT = n VG$. And let $XC = a$, $ng = e$, $cC = o$. Then $VG = \overline{a+o}^{n} = a^{n} + na^{n-1}o + n \times \frac{n-1}{2} a^{n-2} oo + n \times \frac{n-1}{2} \times \frac{n-2}{3} a^{n-3} o^{3} = P - Qo - Roo - So^{3}$. Whence $Q = -na^{n-1}$. $R = -n \times \frac{n-1}{2} a^{n-2}$. $S = -n \times \frac{n-1}{2} \times \frac{n-2}{3} \times a^{n-3}$. Therefore the denſity in G is as $\left(\frac{S}{R\sqrt{1+QQ}} = \right) \frac{n-2}{3a\sqrt{1 + n^{2}a^{2n-2}}}$, that is, as $\frac{1}{\sqrt{a^{2} + nna^{2n}}}$, or as $\frac{1}{\sqrt{a^{2} + nnee}}$, that is reciprocally as $\sqrt{XC^{2} + TZ^{2}}$. And the ſame may be found by fluxions, putting Q, R, S, for the 1ſt, 2d, 3d, fluxion of VG or XC^{n}.

S E C T. III.

[Pr. 12, and inverſly as the velocity; for the time, of deſcribing a given ſpace, is reciprocally as the velocity.

[Pr. 13. caſe 1, 2, 3, the decrement or increment PQ] in a given time.

[ib. cor. and triangle do.] therefore *ex equo*, &c.

Fig. 39. [ib. Sch.] For draw the indefinite line BAP, and make BD perp. and equal to BA; and draw DF, AF parallel to BA, BD. Let AP be the velocity, $AP^2 + 2BAP$ the reſiſtance, AB^2 the force of gravity. Draw DTP, cutting FA in T, and the time of the whole aſcent will be as the triangle DTA.

For draw DVQ cutting off PQ the moment of velocity, and DTV the moment of the triangle; then the decrement of the velocity PQ will be as the reſiſtance and gravity, $AP^2 + 2BAP + AB^2$; that is as BP^2. But the area DTV is to the area DPQ, as DT^2 to DP^2, or as DF^2 to BP^2; therefore ſince the area DPQ is as BP^2, the area DTV will be as the given quantity DF^2. Therefore the area ADT decreaſes uniformly as the time, by the ſubduction of given particles DTV; and therefore is proportional to the whole time of aſcent.

[Pr. 14. From the area DET] This ſhould be; from the moment KLON, ſubduct DTV or $m \times BD$ the moment of DET; and is only a falſe tranſlation.

[ib. cor. or as V^2;] for the ſpace is $\propto \square$ of the time, that is, $\propto$ the ſquare of DET or of $BD \times M \propto$ (becauſe $M = \frac{DE \times V}{DA}$) ſquare of $\frac{BD \times DE \times V}{DA}$, that is (becauſe BD, DE, DA are given) as V^2, or as $\frac{BD \times V^2}{AB}$.

[ib. Sch. inſtead of the uniform reſiſtance made to an aſcending body;] this differs from gravity, only in this; that it cannot generate any motion; but it acts after the ſame manner in all moving bodies in deſtroying their motion, as gravity does in deſtroying the motion of aſcending bodies. Gravity acts uniformly in a given direction. The force

Fig. ariſing from tenacity acts uniformly, but always in a direction contrary to the motion of the body. And therefore when the body is at reſt, it can induce no change in it. Now in the horizontal motion of a body in a fluid, which is reſiſted in part uniformly, one may ſubſtitute the force of gravity for that uniform reſiſtance, as in Pr. 8, 9; 13, 14 And in the aſcent or deſcent of the body in a fluid, inſtead of the force of gravity, one may ſubſtitute the ſum or difference of that uniform force and the force of gravity, as in theſe propoſitions.

SECT. IV.

In Prop 15.

[And the decrements of theſe arcs ariſing from the reſiſtance, will be as the ſquares of the times in which they are generated ;] for ſince the arcs are infinitely ſmall, the reſiſtance affects the ſame as an uniform centripetal force would do

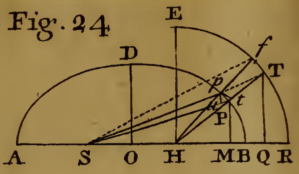
40. [ib. The decrement of the arch PQ will be] for let q, t be the points it would arrive at, in the ſame times, in free ſpace, then $Pq = qt$, and $PQ = Qr$, therefore $Qq = qt - Qr$; and adding Qq, $2Qq = Qt - Qr = tr$; and $Qq = \frac{1}{2}tr$: but $Qq = \frac{1}{4}Rt$; therefore $tr = \frac{1}{2}Rt = Rr$, and $Qq = \frac{1}{2}Rr$.

The ſame demonſtration of this Prop. holds good, as well in aſcending as in deſcending motion.

41. [Cor. 1. The velocity, &c.] for ſuppoſing PQ an arch of a circle whoſe center is ſ; then if PQ or PT be as the velocity in both curves, TQ will be as the centripetal force in both, but in both curves $TQ = \frac{\overline{PQ}^2}{2P\text{ſ}}$; therefore $P\text{ſ}$ = rad. of the circle deſcribed with the ſame velocity.

[Cor.

Fig. 24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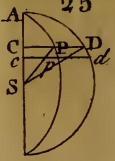
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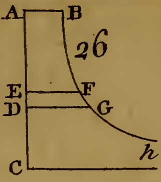
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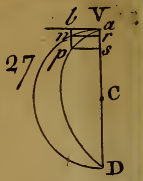
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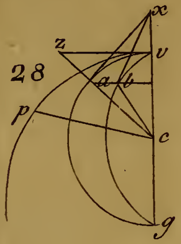
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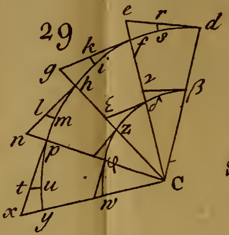
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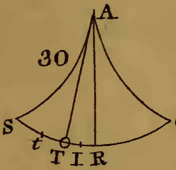
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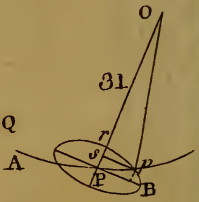
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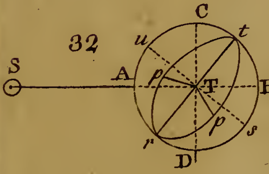
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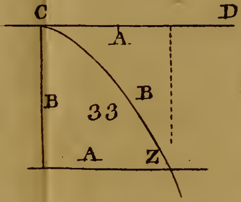
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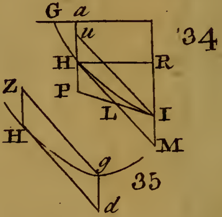
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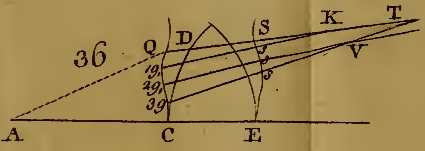
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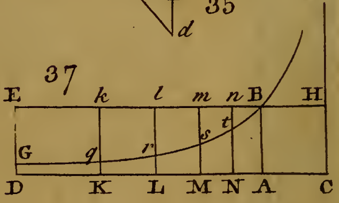
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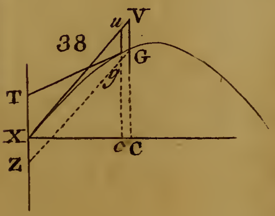
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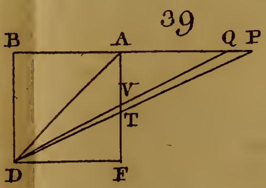
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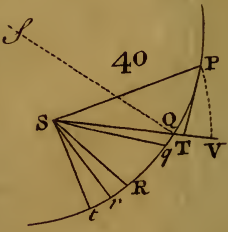
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[Cor. 4. the body will deſcend, &c.] for by cor. Fig. 1. the velocity in P in a reſiſting medium is = velocity in a circle at diſtance SP in free ſpace; but velocity at diſtance SP in free ſpace : to velocity at diſt. $\frac{1}{2}$SP in free ſpace (by cor. 6. Prop. 4. B. I.) or Prop. 34. : : as $\sqrt{\frac{1}{SP}}$: to $\sqrt{\frac{1}{\frac{1}{2}SP}}$: : that is, $\sqrt{1} : \sqrt{2}$; therefore, &c.

[Cor. 7, or as $\frac{2}{3}$AS to AB] (for this ſee my Geometrical Proportion, prop. 24.)

In Prop. 16.

[And the reſiſtance, &c.] inſtead of this proportion in prop. 15, viz. $PQ : QR :: SQ : \sqrt{SP \times SQ}$, ſubſtitute this $PQ : QR :: (SQ^{\frac{n}{2}} : SP^{\frac{n}{2}} ::) \; SQ : SQ + \frac{n}{2} SP - \frac{n}{2} SQ$, (by prop. 24, of my Geomet. Proportion) that is, as the deſcribing velocities, &c.

[Cor. 1.] For the reſiſtance : centripetal force : : $(\frac{1}{2}Rr =) \; \frac{\overline{1-\frac{1}{2}n} \times VQ \times PQ}{2SQ} : (TQ =) \; \frac{\frac{1}{2}PQ^2}{Sp} :: \overline{1-\frac{1}{2}n}\, VQ : PQ :: \overline{1-\frac{1}{2}n}\, OS : OP$.

SECT. V.

In Prop. 19.

[Caſe 3, different ſpherical parts have equal preſſures,] that is, different ſpherical parts of equal magnitude.

In Prop. 20.

[And by a like reaſoning, &c.] Suppoſing the thickneſs of the orbs to be reciprocally as the force of gravity or the denſity of the fluid, or in the complicate ratio of them; and it will appear (as before)

Fig. before) that the bottom is preſſed with a cylinder, of the ſame fluid; whoſe baſe is = the bottom, and height that of the fluid, by the ſame reaſoning as before, and prop. 19.

In Schol Pr. 22.

[By a like reaſoning, &c.] Let the force of gravity be reciprocally as the *n*th power of the diſtance. The ſpecific gravities at A, B, C, &c. will $\propto \frac{AH}{SA^n}, \frac{BI}{SB^n}, \frac{CK}{SC_n}$, &c. and the denſities $\propto$ ſums of the preſſures $\propto \frac{AH \times AB}{SA^n}, \frac{BI \times BC}{SB^n}, \frac{CK \times CD}{SC^n}$, &c. $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}, \frac{CK}{SC^{n-1}}$, &c. And *tu*, *uw*, &c. $\propto$ differences of the denſities $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}$, &c. And $tu \times tb$, $uw \times ui \propto \frac{AH \times tb}{SA^{n-1}}, \frac{BI \times ui}{SB^{n-1}} \propto \frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \propto \frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}}, \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, &c. and therefore univerſally $\frac{1}{SA^{n-1}} - \frac{1}{SE^{n-1}} \propto$ hyperbolic area *thmy*. Therefore if $\frac{SA^n}{SA^{n-1}}, \frac{SA^n}{SB^{n-1}}, \frac{SA^n}{SC^{n-1}}$, or $\frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \frac{1}{SC^{n-1}}$, be ∺; then $\frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}} = \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, and the areas proportional thereto will be equal, viz. *thiu* = *uikw*, and therefore S*t*, S*u*, S*w*, that is, AH, BI, CK ∺. *Q. E. D.* then putting *n* ſucceſſively = 3, 4, 0 — 1, &c. the truth of the ſaid Scholium will appear. And ſo on, *ad infinitum*.

[Other laws of condenſation, &c.] Suppoſe the compreſſing force $\propto$ rth power of the denſity. Then the ſpecific gravities in A, B, &c. $\propto$ $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, &c. And the preſſures in A, B, &c. $\propto \frac{AH}{SA}+\frac{BI}{SB}+\frac{CK}{SC}+\frac{DL}{SD}$ &c, $\frac{BI}{SB}+\frac{CK}{SC}+\frac{DL}{SD}$, &c. Now if theſe ſums were ∺, their differences $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. would be in the ſame ∺, and then the preſſures in A, B, &c. $\propto \frac{AH}{SA}$, $\frac{BI}{SB}$, &c. $\propto \overline{\text{denſities}}^{r} \propto AH^{r}$, BI^{r}, &c. And therefore AH^{r-1}, $BI^{r-1} \propto \frac{1}{SA}$, $\frac{1}{SB}$. And AH, BI $\propto \frac{1}{SA^{\frac{1}{r-1}}}$, $\frac{1}{SB^{\frac{1}{r-1}}}$. Then putting $r=\frac{4}{3}$, $\frac{5}{3}$, 2, &c. all things will appear as in this Schol. But now the aforeſaid ſums are not in geometrical progreſſion, becauſe it is plain their differences are not, viz. $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. for if $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. ∺, and ſeeing SA, SB &c. are ∺, therefore (by Prop. 18. cor. 2. of my Geom. Proportion) AH, BI, &c. would be ∺ which is falſe (except in the caſe of Prop. 21.) therefore neither theſe differences, nor ſums are ∺. And conſequently the concluſions of this Schol. in this reſpect are not true. And therefore it muſt be preſumed, that the Author meant them only to be nearly, and not perfectly true.

Fig.

In Schol. Pr. 23.

[All theſe things are, &c.] It ſeems to me that the prop. holds true, tho' the centrifugal force be extended to all diſtances, in any given law, as well as when it ends at the next particle in the ſame given law. For ſuppoſing the particles of the fluid to be placed in parallel planes, and that theſe planes act on each other; the force of compreſſion on any one plane will be made up of the ſeveral forces, of all the other equidiſtant parallel planes. Therefore ſince the diſtances of all theſe planes before compreſſion are reſpectively proportional to their diſtances after compreſſion from this given plane; their actions on the given plane before and after compreſſion will alſo be proportional, (ſeeing their actions or forces are as ſome power of the diſtances); and their ſums will be alſo proportional to thoſe of any two correſponding planes, viz. as the neareſt to the neareſt. In theſe ſort of particles there will be required a greater force to produce an equal condenſation of an equal quantity of the fluid. But the denſity will be proportional to the compreſſion, as before.

SECT VI.

In Prop. 24.

Cor. 5. —— for let m, M = quantities of matter, w, W = weights, t, T = times, for the common pendulum l. Alſo L = new pendulum. T = new time, for M, and W. Then m : M : : wtt : WTT (by the Prop.) : : $\frac{wtt}{l}$: $\left(\frac{wtt}{l} =\right)$ $\frac{\mathrm{WTT}}{\mathrm{L}}$. And if m, M; t, T, are equal; $w \propto l$, which is cor. 4.

In Prop. 27.

[If the reſiſtance in the arc B, &c.] If the reſiſtance in the arc A were to the reſiſtance in the arc B as AA to AB, the times would be equal, and therefore reſiſtance AA in the arc A cauſes the exceſs of time above that in a non-reſiſting medium; and reſiſtance AB in the arc B cauſes the exceſs of time in B above that in a non-reſiſting medium, equal to the former exceſs, (becauſe with theſe reſiſtances) they are deſcribed in equal time. But now if B be deſcribed with a reſiſtance BB; the exceſs of time will be caus'd by that reſiſtance.

[But thoſe exceſſes, &c.] Now the exceſſes of time cauſed by deſcribing B with the reſiſtances AB, BB reſpectively, are as the ſpaces left undeſcribed after the given time; that is, as the reſiſtances AB, BB nearly, or as A to B, viz. as the arches A, B. And ſince the exceſs of time by deſcribing A with reſiſtance AA = exceſs of time by deſcribing B with reſiſtance AB. Therefore it follows, that exceſs of time in A : to exceſs of time in B : : as A : to B nearly.

In Prop. 29.

[Area PIEQ may be to the hyperbolic area PITS : : as BC to C*a*; and that the area IEF may be to the area ILT as OQ to OS.]

That this is poſſible (aſſuming P at pleaſure) may be thus ſhewn. Let the areas PIEQ, PITS be taken (as BC to C*a*) indefinitely ſmall, then PIEQ muſt be greater or at leaſt equal to PITS; ſuppoſe them equal, then will PIFQ > PILS, and IF > IL (and more ſo, if PIEQ > PITS), but when S, P, Q coincide OS to OQ is a ratio of equality, therefore in this caſe IFE to ILT (or IF to IL) > OQ to OS (or however not leſs), increaſe the areas PIEQ and PITS in the ſame ratio, and the

Fig. the ratio of OQ to OS will alſo increaſe and converge at length to the ratio of IEF to ILT; which ſhews the poſſibility of what was required; (but as to an actual ſolution I ſhall refer it to an algebraic proceſs.) This being done as required; then, the points O, S, P, Q are all fixt, and the point R only variable.

[And the increment, &c.] for, the fluxion of $\frac{IEF}{OQ}$ OR — IGH to the fluxion of — PIGR : : as $\frac{IEF}{OQ}$ — HG to — RG : : or as HG — $\frac{IEF}{OQ}$ to RG. Or rather thus, the increment of $\frac{IEF}{OQ}$ OR—IGH, is as $\frac{IET}{OQ} - HG \times - Rr = HG \times Rr - \frac{IEF}{OQ} \times Rr$. And the increment of PIGR is as $RG \times - Rr$; and therefore the decrement (becauſe the decrement is a negative increment) of PIGR, is as $RG \times Rr$.

In cor. to Pr. 30.

[Now if the reſiſtance DK be in the duplicate, &c.] for the velocities being as the ordinates of a circle or ellipſis deſcribed on *a*B; the reſiſtances (being as the ſquares of theſe velocities) are as the ſquares of theſe ordinates, that is, as the rectangles of the abſciſſa's. Therefore the reſiſtances DK, QV are reſpectively as *a*DB, (*a*OB =) OB^2. But this is the property of the parabola *a*VB deſcribed to the axis VO, by cor. 2. to p. 4. of the parabola.

Gen. Sch. Pr. 31, pa. 96.

[But in leſſer oſcillations ſomewhat greater, &c.] that is, the difference in the leſſer arc to the difference in the greater arc, is in a greater ratio than the ſquares of theſe leſſer and greater arcs.

[pa. 97.] for $\frac{1}{2}$ the cords of theſe arcs, read the Fig. cords of thoſe half arcs.

[ib. p. 98. As $\frac{7}{11}\,AV + \frac{7}{10}\,BV^{3} + \frac{3}{4}\,CV^{2}$.] Theſe three members are taken to denote three laws of reſiſtance, viz. the firſt in the ſimple ratio of the velocity, the third in the duplicate ratio thereof, and the ſecond partly in the ſimple partly in the duplicate ratio (or in a mean between them); and $\frac{7}{10}$ the coefficient thereof is a middle one between $\frac{7}{11}$ and $\frac{3}{4}$.

[As 0,041748 to 121] it ſhould be as 0,041778 to 121.

[pa. 99. It is manifeſt that the force, &c.] for the forces of reſiſtance and gravity are as their effects, to wit, as the velocity loſt (by reſiſtance) and velocity gained (by gravity) in the ſame time, as 1 to $376\frac{1}{50}$.

[I alſo counted——I leave the calculation, &c.] the calculation after the preceding manner will be as follows.

Mean oſcillations —	$3\frac{1}{2}$	7	14	28	56	112
diff. between 1ſt deſ. and laſt aſ.	$\frac{1}{2}$	1	2	4	8	16
diff. of arcs deſcribed in the deſcent and ſubſequent aſcent.	$\frac{1}{748}$	$\frac{1}{272}$	$\frac{4}{325}$	$\frac{12}{250}$	$\frac{24}{125}$	$\frac{48}{68}$

Alſo theſe in the greater arches are nearly in the duplicate ratio of the velocities, but in the leſſer arches ſomewhat greater, (theſe arcs are in the laſt ſeries of the table).

Let V = velocity in 2d, 4th, and 6th caſe = 1, 4, 16 reſpectively. And the difference of the arcs will be, in the

2d Caſe $= \frac{1}{272} = A + B + C$
4th Caſe $= \frac{12}{250} = \frac{6}{125} = 4A + 8B + 16C$
6th Caſe $= \frac{48}{68} = \frac{12}{17} = 16A + 64B + 256C$.

whence $A = ,0005098$; $B = ,0005882$; $C = ,0025784$. Therefore the difference of the

arcs

Fig. arcs is as $,0005098V + ,0005882V^{\frac{3}{2}} + ,0025784V^2$. And conſequently the reſiſtance of the globe in the middle of the arc, will be to its weight (by cor. pr. 30.) as $,00032442V + ,00041174V^{\frac{3}{2}} + ,0019338V^2$, to 121. Therefore the reſiſtance will be to the weight, in caſe 2d, 4th, and 6th, as ,00267 to 121; ,0355324 to 121; ,5265949 to 121, reſpectively.

Note, you may take any other numbers (that are in the ſame proportion with theſe above) for V; for example I took $\frac{1}{4}$, 1, and $4 = V$, in the 2d, 4th and 6th caſes, and A will be $= ,002039$; $B = ,004706$; $C = ,041255$; and the reſiſtance to the weight (at the laſt) comes out the ſame as before.

In the 6th caſe the point marked in the thread deſcribed an arc of $112 - \frac{48}{68}$ inches $= 111\frac{5}{17}$; therefore the center of the globe deſcribes an arc $= 115\frac{167}{187} = 115\frac{8}{9}$ nearly; and its velocity is (nearly) the ſame as in deſcending an arc $57\frac{17}{18}$ of a cycloid (whoſe ſemi-arc is 126,) or = velocity acquired by falling perpendicularly thro' the verſed ſine (or abſciſſa) of that arc $57\frac{17}{18}$, but this verſed ſine is = 13,324. Therefore the velocity of the pendulum is = velocity acquired by falling perpendicularly 13,324 inches. And with this velocity, the globe meets with a reſiſtance which is to its weight as ,52659 to 121, or (taking that part only of the reſiſtance which is as the ſquare of the velocity,) as ,49505 to 121.

Alſo if a globe of water of equal magnitude moves with the ſame velocity, its reſiſtance will be to its weight as ,49505 to 213,4; or as 1 to $431\frac{1}{15}$. Whence, in the time in which the globe uniformly deſcribes 26,648 inches, the weight of the globe of water will generate all that velocity in the falling globe; therefore the velocity deſtroyed

by reſiſtance, will be to that acquired by gravity Fig.
(in the ſame time), as 1 to $431\frac{1}{15}$, or velocity loſt $= \frac{1}{431\frac{1}{15}}$ of the whole velocity. And therefore in the time it would uniformly deſcribe its ſemidiameter, with the ſame velocity, it would loſe the $\frac{1}{3341.7}$ part of its motion.

[pa. 101,2, after five oſcillations] this certainly ſhould be after 10 oſcillations, as appears by the proceſs of the calculation.

[pa. 102. So that the difference 0,4475, &c.] for the motion loſt is $\propto$ the reſiſtance, and that $\propto$ the ſquare of the velocity and ſquare of the time nearly, which in this caſe is a conſtant product.

[pa. 103, l. 6.] Here is a ſmall error, viz. ,61675 inſtead of ,61705; whence 45,453 would have been 45,43; and the reſiſtances as 7,002 to 1; but this is not material.

[pa. 104. becauſe the reſiſtance, &c.] for the reſiſtance $\propto$ ſquare of the time × ſquare of the velocity, which product is invariable, becauſe equal arches are always deſcribed.

SECT. VII.

[Pr. 32. ratio of denſity] he means any given ratio of denſity, which are proportional to the particles.

[Pr. 32. Proportional times] any times, in that conſtant ratio, that is, let them move among one another, in ſimilar directions, with velocities which are as the particles.

[Pr. 32. cor. 2. ſpaces proportional to their diameters] in theſe proportional times.

[Pr. 33. cor. 2. for if the forces, &c.] for the reſiſtance ariſes from the centrifugal forces, and from the colliſions, of the particles. The reſiſtances of the firſt ſort are as the motive forces,

E that

Fig. that is, as the accelerative forces and quantity of matter, that is, (by ſuppoſition) as the ſquares of the velocities and quantities of matter, that is, (becauſe the quantities of matter are given, the fluid being the ſame) as the ſquares of the velocities.

The reſiſtances of the ſecond ſort are as the number of reflexions, and their forces; that is, (as is proved in the Prop.) as the ſquares of the velocities, ſquares of the diameters, and denſities of the parts, that is (becauſe the diameters and denſities are given) as the ſquares of the velocities, accurately.

[As alſo the bodies E and G;] let E and G be vaſtly ſwifter than D and F.

[Pr. 34. but the former of, &c.] for $bH \times CB = (BE^2 =) ObN$, which is the property of the parabola.

[Pr. 34. Schol. leſs reſiſted than the former ſolid;] for, if FG and IH be produced till they interſect, they will form a right angle; and therefore the fruſtum FGBHI will be leſs reſiſted than if the lines ſhould interſect in an obtuſe angle, (as is ſhewn in the cone;) which they would do if the lines fell within FG and IH; and for the ſame reaſon the reſiſtance would be greater in the curve itſelf, becauſe the lineolæ which conſtitute it, will, when produced, interſect one another at obtuſe angles. What is mentioned beſides in this Schol. is demonſtrated in the appendix.

[Pr. 35. it follows, that the cylinder, &c.] for mot. cylinder : mot. medium : : cylinder's magnitude × denſity × velocity : (medium's mag. × denſ. × vel. =) cylinder's $\frac{\text{magn.}}{2}$ × 2 vel. × medium's denſity : : cylinder's denſity : medium's denſity.

[ib. to the force by which] to the motive force by which its whole motion, &c.

[Pr. 35. cor. 2.] for, the reſiſtance of the globe and the force (that will quite take away its motion in the time it moves two or four thirds of its diameter,)

ameter,) are in a given ratio; therefore increaſe Fig. the velocity of the globe in any ratio, and the force (that will utterly deſtroy all its motion in a given ſpace) will be increaſed in a ratio of the velocity directly and the time inverſely, that is, as the ſquare of the velocity, therefore the reſiſtance of the globe is increaſed in the duplicate ratio of the velocity.

[ib. cor. 3.] for the ſame reaſon, increaſe the diameter of the globe in any ratio, and the motive force (that will utterly deſtroy its motion) will be increaſed in the triplicate ratio of the diameter directly, and the time inverſely, that is, in the triplicate ratio of the diameter directly, and the ſimple ratio of the diameter inverſely, that is, as the ſquare of the diameter; and therefore the reſiſtance of the globe will be encreaſed in the duplicate ratio of the diameter.

[Cor. 6. and its reſiſtance, &c.] Let BC be the reſiſtance at firſt, then reſiſtance at the firſt : reſiſtance at the end : : $BC^2 : EF^2$; but AE : AB : : BC : EF : : EF : BH; therefore $BC^2 : EF^2$: : BC : BH. ∴ BH is the reſiſtance at the end.

[Cor. 7. As the logarithm, &c.] For (by Schol. 42. Prop. 86, hyperbola,) it appears, that if $AH = 1$. $AB = T$. $BK = t$: then it will be CBKF : EHKF : : ($\log. T+t - \log. T =$) $\log. \frac{T+t}{T} : \log. \overline{T+t}$.

And EHKF : AHED : : ($\log. \overline{T+t}$: ,43429448 : :) $\log. \overline{T+t} \times 2{,}30258509 : 1$. Alſo AHED : CBKG : : HE : $CB \times t$: : $T : t$. Therefore *ex equo*, CBKF : CBKG : : $\log. \frac{T+t}{T} \times 2{,}30258 \times T : t$: : $\log. \frac{T+t}{T} \times 2{,}30258509 : \frac{t}{T}$.

[Pr. 36. cor. 2.] Forces are equal when their effects are equal in a given time; but the effect of the motion of the effluent water (in the time a bo-

Fig. dy deſcends thro' GI) = a cylinder, whoſe length is 2GI, and baſe the hole EF; alſo the effect of the gravity or weight of a column whoſe height is GI, and baſe the hole EF = (by reaſon of an accelerated motion, and in the ſame time) a cylinder whoſe length is GI, and baſe the hole EF; and therefore the effect of the preſſure of twice that column (in the ſame time) = a cylinder, whoſe length is 2GI, and baſe the hole EF; = the firſt cylinder, *ergo*, &c.

[Cor. 3. or IH + IO to 2IH.] for $ib : ho :: io : og$; or $ho : og :: ib : io$. And $ho + og : 2ho :: ib + io : 2ib :: io + ig : 2io :: \odot EF + \odot AB : 2\odot EF$.

[ib. cor. 10.] This cor. ought to be more exactly computed and demonſtrated, becauſe the following propoſitions depend thereon.

[Pr. 37. pa. 137. And is therefore nearly equal, &c.] This reſiſtance = weight of a cylinder (whoſe baſe is that little circle and altitude $\frac{1}{2}$IG, from which altitude the cylinder muſt fall, &c.) = force by which its motion may be generated, &c.

[Pr. 37. cor. 1.] Let F = force (mentioned in the prop.) M = medium's denſity. C = cylinder's denſity. V = velocity. D = diameter. T = time (of deſcribing four times its length). R = Reſiſtance. Then $R = F\frac{\times M}{C}$ (by the prop.) $\propto C \times \frac{V}{T} \times D^2 \times \frac{M}{C} \propto V^2 D^2 M$.

[Cor. 2.] Let EF be given, and denſity of the medium be alſo given. Then becauſe this reſiſtance = weight of a cylinder (whoſe baſe is PQ, and height $\frac{1}{2}$IG) $\times \frac{EF^2}{EF^2 - \frac{1}{2}PQ^2}$; therefore weight of this cylinder = reſiſtance $\times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2}$. But the force requiſite to take away the motion of this cylinder, whilſt it moves four times its diameter Fig. $\propto$ weight of this cylinder $\times$ velocity $\times$

$$\frac{1}{\text{time of deſcribing 4 times its length}} \propto \text{reſiſtance} \times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2}.$$

And the denſity of the cylinder being increaſed, the force is increaſed in the ſame ratio.

[Cor. 3.] for ſince ref. : force : : $EF^6 \times$ denſity medium : $\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2 \times$ denſity cylinder : : med. denſity : $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6}$ $\times$ cyl. denſity ; increaſe the diſtance it moves, in any ratio ; and the force (to deſtroy the motion) will be decreaſed in that ratio, as ſuppoſe the ratio of $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6}$. Then ref. : force : : med. denſity : cylin. denſity.

[Sch. pr. 37. AE and BE be 2 parabolic arcs] 43. ſince the reſiſtance is the ſame, whether the fluid or the cylinder move, therefore the caſe of the reſiſtance of the circle in fig. 4. is the ſame caſe with the reſiſtance of the circle AB in fig. 6, conſidering the velocities HG and EG, wherewith they are acted on ; but (by cor. 9, pr. 36.) the reſiſtance on PQ $= \frac{1}{2}$ cylinder on PQ and height GH (*thro'* which a falling body acquires the velocity, which the fluid has on PQ), but $\frac{1}{2}$ this cylinder is $=$ a parabolic ſpindle on PQ and height GH, nearly. Therefore the reſiſtance on AB $= \frac{1}{2}$ cylinder on AB and height GE ($=$ GH,) thro' which a falling body acquires the velocity with which the cylinder moves,) which is equal to a parabolic ſpindle on AB and height GE $=$ AEB by conſtruction, (ſince lat. rect : ($HG =$) GE : : GE : AG.) This is the very conſtruction of fig. 2.

Fig. [ib. muſt be to this force as 2 to 3, at leaſt,] for the weight upon the little circle (in cor. 7. p. 36.) is there ſhewn to be equal to a cylinder of $\frac{1}{3}$ the height GH at leaſt. But the weight of a cylinder on that circle and $\frac{1}{2}$ that height is the force whereby the cylinder's motion may be generated in the time it moves 4 times its length or 2GH; therefore the reſiſtance in that cor. is to the weight here demonſtrated (*ceteris paribus*) that is, to the force : : as $\frac{1}{3}$ to $\frac{1}{2}$: : that is as 2 to 3, at leaſt; and increaſing the denſity, the force is increaſed in the ſame ratio.

[ib. let CF and DF.] It is neceſſary to allow as much time for the meeting of the particles of water (at the axis of the ſolid from the ſide thereof), after they are paſt the ſolid, as for their ſeparation before they come to it. Now ſuppoſe this to be the caſe of fig. 4. In the time that a particle paſſes from H to P, the ſame particle with the velocity in P, will in the ſame time deſcribe 2HG, and therefore muſt paſs twice as far to arrive to the axis again. Whence in fig. 6, the particle F muſt be twice as far from CD, as E from AB; but the height of FCD = 2 height of AEB, by conſtruction. For let z, y = 2 ordinates to $x = \frac{1}{2}$AB; and $zz = 4rx = 4yy$; and $z = 2y$.

[Cor. 1. Pr. 38.] the demonſtration is the ſame with that of cor. 1. pr. 37.

[Cor. 2. Pr. 38.] Let denſity of globe : denſ. of fluid : : d : 1. Then $d \times \frac{4}{3}$ diameter of the globe = ſpace fallen, and $d \times \frac{8}{3}$ diam. globe = ſpace it deſcribes afterwards, in the time of its fall. Then, becauſe forces (which generate a given motion), are reciprocally as the (times, that is reciprocally as the) ſpaces which any body deſcribes with the ſame uniform velocity. Therefore, the force of the globe's comparative weight, (which generates its motion, in the time it deſcribes $d \times \frac{8}{3}$ diam.) : force that generates the ſame motion, (while it deſcribes $\frac{8}{3}$ diam.)

$\frac{8}{3}$ diam.) : : $\frac{8}{3}$ diam. : $d \times \frac{8}{3}$ diam. : : 1 : d : : denſity of the fluid : denſity of the globe. Therefore, &c. Fig.

[Cor. 3. Pr. 38.] The velocity being given, the motion M is alſo given; but (by this Prop.) reſiſtance × d = force, which will generate the motion M (in the time of moving $\frac{8}{3}$ diameter); but this force muſt be = reſiſtance, which will quite take away the ſaid motion M (in the ſame time of moving $\frac{8}{3}$ diameter). Now augment the time of moving (preſerving the velocity) in any ratio, (let it = T.) And this laſt reſiſtance (which will quite take away the ſaid motion M, in the time T) will be decreaſed in the ſame ratio (= R). Therefore T and R are alſo given. Therefore by cor. 7, &c.

[Cor. 4. Prop. 38.] for let M = $\frac{8}{3}$ diameter in the time T; and becauſe (by ſup.) $\frac{tM}{T+t} = \frac{1}{2}M$, therefore $T = t$. Now (by cor. 7. pr. 35.) it will be$\left(\frac{t}{T} =\right) 1 : (\log. \frac{T+t}{T} \times 2, 3,$ &c. $= \log. 2 \times 2, 3,$ &c. $=)$,693 : : $\frac{8}{3}$ diameter : $\frac{5{\cdot}545}{3}$ diameters, which is leſs than two diameters. Now that M ought to be = $\frac{8}{3}$ diameter in the time T is plain; for it is required (by cor. 7. pr. 35.) that the reſiſtance R quite takes away M in the time T. But (by this prop. 38.) the reſiſtance (= force required) will juſt take away the motion M in moving $\frac{8}{3}$ diameter. But if T be any other time than that of moving $\frac{8}{3}$ diameter, then the force to deſtroy M in that time will not be = reſiſtance, but greater or leſſer than it.

[Prop. 40. let the globe, &c.] This is calculated in Prob. XIV. of my book of Fluxions. In cor. 2. of that Prob, G, H and N are found, and thence the velocity $= \frac{N-1}{N+1}$ H, where H is the greateſt velocity the globe can acquire, in falling in the fluid; and the height fallen comes out the ſame in value with his, but in other ſymbols; but the one may be eaſily reduced to the other.

[ib. the numbers in the fourth column, &c.] 2F is the ſpace deſcribed in the time G with the greateſt velocity H, (by the Prob.) therefore, G : 2F : : P : $\frac{2P}{G}$F = the ſpace deſcribed in the time P with the greateſt velocity, the numbers in the fourth column.

[Sch. Pr. 40. and any other globe, &c.] that is, the magnitude of any body $\propto$ the exceſs of its weight in vacuo above that in water. Or 132,8 : 1 : : this ſaid exceſs : magnitude of the globe. Hence, Diameter globe (in Exp. 1.) = 0,84224.

[ib. Exp. 1. but this ſpace, by reaſon, &c.] to demonſtrate this, it is neceſſary to premiſe, that, the greateſt velocity wherewith a globe can deſcend by its comparative weight thro' a fluid in a canal, is that which it may acquire by falling with the ſame weight, without reſiſtance, and in its fall deſcribing a ſpace which is to $\frac{4}{3}$ diameter as mn^2d, to l^3. Puting denſity globe : denſity fluid : : d : 1. and l = orifice of the veſſel. N $= l -$ great circle of the globe, $m = l - \frac{1}{2}$ great circle. For (as in cor. 2, Pr. 38.) the ſpace it afterwards deſcribes (in the ſame time of its fall) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter; then the force of the globe's comparative weight (which generates its motion, in the time it deſcribes $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter) : force (that generates the ſame motion in the time it deſcribes $\frac{8}{3}$ diameter) : : ($\frac{8}{3}$ dia-

($\frac{8}{3}$diameter : $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter : :) $l^3 : mn^2d$; therefore (by Prop. 39.) the reſiſtance of the globe = force of the globe's comparative weight (which generates this motion in the time it deſcribes $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter, or falls $\frac{mn^2d}{l^3} \times \frac{4}{3}$ diam.) And therefore this force cannot accelerate the globe. *Q. E. D.* Fig.

Now,

By the experiment; $2F = d \times \frac{8}{3}$ diameter; but the ſpace the globe uniformly deſcribes (with the greateſt velocity wherewith it can deſcend in the canal or veſſel, and in the time of acquiring it with its compound weight) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter $= \frac{mn^2}{l^3} \times 2F = \frac{mn^2}{l^3} \times 4.4251$. Alſo $\sqrt{95{,}219} : \sqrt{\frac{mn^2}{l^3} \times F} :: 1'' : \sqrt{\frac{mn^2}{l^3}} \times \sqrt{\frac{F}{95{,}219}} = \sqrt{\frac{mn^2}{l^3}} \times G = \sqrt{\frac{mn^2}{l^3}} \times 0{,}15244$ (by the laws of falling bodies) = time of acquiring the greateſt velocity. And therefore ($\sqrt{\frac{mn^2}{l^3}} \times G$ (=time) : $\frac{mn^2}{l^3} \times 2F$ (= ſpace) : :) $G : \sqrt{\frac{mn^2}{l^3}} \times 2F :: 4'' : \frac{n}{l}\sqrt{\frac{m}{l}} \times \frac{8F}{G}$ = (ſpace deſcribed in $4''$) $= \frac{n}{l}\sqrt{\frac{m}{l}} \times 116{,}1245$. Subduct $\frac{mn^2}{l^3} \times 1.3862944F = \frac{mn^2}{l^3} \times 3{,}0676$. And $\frac{n}{l}\sqrt{\frac{m}{l}} \times 116{,}1245 - \frac{n}{l}\sqrt{\frac{m}{l}} \times 3{,}0676$ = ſpace fallen in the fluid, $= 116{,}1245$

— 3,0676 × $\frac{n}{l}\sqrt{\frac{m}{l}}$, nearly, as the author makes it.

[ib. ex. 13.] This and the following are upon the ſame computation as the foregoing experiments in water, to wit, the theory of non-elaſtic fluids; the reaſon is, becauſe he conſiders fluids as elaſtic, whoſe parts are diſcontinued, &c. ſee Sch. Pr. 35.

[ib. pa. 160. equal to $\frac{1}{4586}$, &c.] This is plain from what follows, afterwards. For let a ſpace : $\frac{8}{3}$D : : 860 : 1; this ſpace = $\frac{6880}{3}$D = 2293D, and this ſpace the globe moves in the time T, with velocity V; let therefore T = 2293, and $t = \frac{1}{2}$. Then, motion loſt = $\frac{\frac{1}{2}V}{2293 + \frac{1}{2}} = \frac{1}{4587}V$; note t ($= \frac{1}{2}$) is not ſenſibly increaſed, by the reſiſtance, in moving thro' $\frac{1}{2}$D; and hence I have taken it for the ſame.

[ib. let D be the, &c.] for, put denſity globe : denſity fluid : : d : 1; alſo let the ſpace $\frac{8d}{3}$D = S. Now by Prop. 38, d × reſiſtance = force, which (uniformly continued) will quite deſtroy its motion, in the time it moves $\frac{8}{3}$D; therefore d × reſiſt. will quite take away its motion, whilſt it moves $\frac{8}{3}$D. But (becauſe the force of that reſiſtance and the time of its acting are reciprocally proportional), the reſiſtance the globe meets with will quite take away its motion whilſt it moves (with V) $\frac{8d}{3}$D = S. Therefore (by cor. 7. Pr. 35.) the globe in the time T (of moving S), meets with a reſiſtance R, which will quite deſtroy its motion V; therefore in the time t, it will loſe $\frac{t}{T+t}$V, &c.

SECT VIII.

Fig. 44.

[Pr. 42. and becauſe the motion of the waves, &c.] Let the waves A, B, C move from A towards C, and ſuppoſe the water to deſcend from the vertex C into the hollow E, and thence up to B, which it will do ſuppoſing the vertices A, B, C to be of equal altitude; alſo let the water deſcend from B to D and aſcend to A, &c. Now ſince the water in the vertex B deſcends towards D, at the ſame time that other water (that had deſcended from C to E,) is aſcending to B; therefore the water will be lower at the point B and higher at the point *b* than before; and conſequently that the vertex of the wave B will be transferred towards C. Therefore the vertices (and hollows and any correſpondent points) of all the waves will move from thence towards C; therefore the waves will be carried on by the deflux of water down CE, BD; and the accumulation thereof, on EB, DA.

[Pr. 43. caſe 1. at nearly equal diſtances, &c.] they ſet forward at equal diſtances, and continue ſo, becauſe they move with equal velocity, as appears by (the demonſt. of caſe 1, 2, 3 of) Prop. 48. 45.

[Pr. 46. becauſe the motion of the waves is carried on, &c.] Let A be the vertex of a wave. The fluid at *b* is more preſſed (by reaſon of a greater depth of incumbent water) than *d*; therefore that greater preſſure cauſes the water to recede from *b* in all directions, and to aſcend at *c*; therefore the vertex of the wave is now in *c*. Alſo a greater preſſure of water propagated from *d* to *f*, cauſes the water to deſcend at *c* and aſcend at *e*, therefore the vertex of the wave moves along *a*, *c*, *e*, &c. And this it will always continue to do, if at firſt any force is ſuppoſed to be applied

to

Fig. to the ſide *gha*, to hinder the aſcent of the water there in like manner as it aſcends at *c*, *e*; for then the ſide *gha* will be afterwards always deſcending, and the ſide *ace* aſcending, without the continuance of ſuch force.

[ib. tranſverſe meaſure] is meaſuring along the ſurface of the water; as is plain from the demonſtration.

46. [ib. cor. 2. rather performed in a circle] as in the note on Prop. 42. But if the breadth of the waves be meaſured, not in the right lines *acb*, but along the ſurface of the water (becauſe the deflux of water thro' *b*, *c*, *a* is made in a curve line near a circle), then the time mentioned in the prop. will be very nearly as it is there aſſigned.

47. [Prop. 47,] fig. 1. pl. 9. is wrong, it ſhould be thus; (and the original is wrong too,) where *l*, *m*, *n*, ſtand at N, M, L.

[ib.—with thoſe of an oſcillating pendulum.]
[ib. according to the law of a pendulum oſcill.]
for, by Prop. 52. lib. 1. ſpace deſcribed by a pendulum, in the time PI; is to a whole vibration; as the time PI, to the time PIS; and to two vibrations, as PI, to PHSP, and the accelerating force of a pendulum, is as its diſtance from the loweſt point. See cor. Pr. 51.

[ib. that is, (becauſe HL — KN is, &c.] that is, HL — KN : HK :: OM : OI.

[Pr. 48. caſe 1. diſtances of the pulſes, &c.] the diſtances of the pulſes in one medium be = diſtances of pulſes in the other.

[ib. and therefore the pulſes;] hence it appears that the correſpondent parts in both mediums, (vibrate or) go and return together, or in equal times, tho' one makes longer vibrations than the other. But (by ſuppoſition) the diſtances of pulſes in both mediums are equal; and the pulſes in both are tranſlated thro' theſe equal diſtances in the Fig. times of the parts of the medium's (vibrating or) going and returning, which times are proved equal; therefore the pulſes in both mediums move equal diſtances in equal times, and therefore are equally ſwift.

[Pr. 48. caſe 2. then will their contractions, &c.] be equal, in equal ſpaces.

[ib. and moreover, &c.] for ſpace $\propto \frac{\text{ſquare of the time}}{\text{matter}}$ (for theſe ſpaces are generated by accelerative forces), and time $\propto \sqrt{\text{ſpace}} \times \sqrt{\text{matter}} \propto \sqrt{\text{ſpace} \times \text{ſpace}} \propto$ ſpace.

[ib. Caſe 3. the time which is neceſſary, &c.] for ſpace $\propto \frac{\square\ \text{time} \times \text{force}}{\text{matter}} \propto \frac{\square\ \text{time} \times \text{force}}{\text{denſity}}$.

Therefore if the ſpace be given, time $\propto \sqrt{\frac{\text{denſity}}{\text{force}}}$.

Therefore velocity $\propto \frac{1}{\text{time}} \propto \sqrt{\frac{\text{force}}{\text{denſity}}}$.

[Prop. 49. — ratio of PO to A conjunctly;] that is, time $\sqrt{\frac{\text{matter}}{\text{force}}} \propto \frac{1}{\sqrt{\text{force}}}$, when the quantity of matter is given, (this in accelerated motions). Then time of one vibration by that elaſtic force : time of one vibration of a pendulum PO, by the weight : : $\sqrt{VV} : \sqrt{PO \times A}$, and time of one vibration of PO, by the weight : time of one vibration of A : : $\sqrt{PO} : \sqrt{A}$. Therefore, &c.

[ib. —of the going and returning] of the pendulum PO.

[ib. — of one oſcillation compoſed of the going and returning,] of the pendulum A.

[ib.

Fig. [ib. cor. 1. — to its circumference.] For (Mechanics, Prop. 24.) diameter : circumference : : time of fall thro' $\frac{1}{2}$ radius : time of one vibration. Therefore as radius : circumference : : time of fall thro' $\frac{1}{2}$ radius : time of two vibrations.

[ib. cor. 2.] for velocity $\propto \frac{\text{ſpace}}{\text{time}} \propto \frac{A}{\sqrt{A}} \propto \sqrt{A} \propto \sqrt{\frac{e.\text{force}}{\text{denſity}}}$. By elaſtic force he means, that which ariſes by heat, as well as that ariſing from the compreſſion.

[Prop. 50. Let the number] of the double vibrations, &c.

[ib. Schol. in the ſubduplicate ratio of the defect of the matter] that is, of the quantity of vapour or watery particles. And (by Prop. 49, cor 2.) the velocity will be increaſed as (the $\sqrt{\text{denſity}}$ that is, as) the $\sqrt{\text{quantity of true air in a given ſpace}}$, is decreaſed.

[ib. near 100 pulſes in] This computation preſuppoſes, that thoſe vibrations of the ſtring are double vibrations, or vibrations compoſed of its going and return.

SECT. IX.

[Prop. 51, but the differences of the angular motions, &c.] theſe are as the difference of the abſolute motions round the axis, that is, as the difference of the abſolute tranſlations, or as the relative tranſlations (which are thoſe here ſpoken of) directly, and the diſtances inverſely.

[ib. of quadratures of curves, &c.] Let SD $= x$; D$d = y$; then D$d \propto \frac{1}{xx}$ $\left(= \frac{b}{xx}\right)$ by conſtruction; therefore $y = \frac{b}{xx}$; and $y\dot{x} = bx^{-2}\dot{x}$; and

$b\dot{x}$

$\frac{bx^{-1}}{-1} = \frac{-b}{x} =$ area; therefore area $\propto \frac{1}{x} = \mathrm{D}d\mathrm{Q}$.

[ib. cor. 3.] In a given time let the parts 1, 2, 3, 48.
of the fluid deſcribe the ſpaces 1*r*, 2*s*, 3*t*; take
away or add the equal angular motions 1*n*, 2*o*, 3*p*;
ſo that the ſpaces *nr*, *os*, *pt* be deſcribed in that
given time; now whether the points 1, 2, deſcribe
the ſpaces 1*r*, 2*s*, or *nr*, *os*; the tranſlations at the
end of the given time will be $= si$; alſo if 2, 3
deſcribe 2*s*, 3*t*, or *os*, *pt*, the tranſlations at the
end of the given time will be the ſame in either
caſe, viz. $= ut$; therefore the tranſlations
being equal and the attrition alſo, the motions
will be continued.

[Pr. 52. cor. 8. about any given axis] paſſing thro' the globe; the relative motion of the parts of the globe and fluid are the ſame ſtill, therefore, &c.

[ib. cor. 9. ſemidiameter of the globe;] let BC 49.
be a plane, D a point of the veſſel, A a point of
the globe; then period. time of A : time B : : $<$ velocity B : $<$ velocity A; and therefore time A
$+$ time B : time A : : (vel. B $+$ vel. A $=$) $<$
vel. A from B : $<$ vel. B : : time B : time A from
the plane BC (to BC again) : : $\mathrm{CB}^2 : \mathrm{CA}^2$. But
time B from DC $=$ time D from BC; therefore,
time D (from BC) : time A (from BC) : : CB^2 :
CA^2; as it ſhould be by the prop. and cor. 8.

BOOK III.

[PROP. 3. the action of the ſun, attracting, &c.] ſee cor. 7. Pr. 66. lib. 1, or cor. 14, where KL is $\propto$ PT nearly, (SK, or ST being given); and the force TM in its mean quantity, is = PT, nearly.

[Prop. 4. and the ſpace which a heavy body deſcribes] by falling in $1''$: half the length of the pendulum (= $\frac{1}{2}$radius) : : ſquare of $1''$: ſquare of the time of falling thro' half the pendulum : : (Mechanics, Prop. 24.) ſquare of the circumference : ſquare of the diameter.

[ib. pa. 216. line 13, the mean diſtance of 60 diameters] it ſhould be 60 ſemidiameters, as it is in the original.

[Prop. 6. p. 222.—ſubduplicate of that proportion, as by ſome computations I have found,] *i. e.* as I found by making ſome calculation.

For let R be the diſtance of the Sun and Jupiter, d the greater force, e the leſſer; then to find the diſtance x where d may be diminiſhed to e. Becauſe the forces are reciprocally as the ſquares of the diſtances, ſay $\frac{1}{R^2} : \frac{1}{x^2} :: d : e :: x^2 : R^2$. And $x : R :: \sqrt{d} : \sqrt{e}$; therefore at the diſtance x, the weight d (of the ſatellite) would become equal to e; and the center of their orbits, would not be in Jupiter, contrary to experience. And if the force of the ſatellite was leſs; by a like computation, the center of their orbits would be nearer than Jupiter, alſo contrary to experience.

[Prop. 8. cor. 1. and $\frac{1}{169282}$ reſpectively] this ſhould be $\frac{1}{196282}$, or rather $\frac{1}{194075}$.

[ib. at the diſtances 10000, &c.] theſe are as the diameters; and the apparent diameters are found by aſtronomical obſervation; which × by the proportioned diſtances, gives the ratio of the real diameters.

[Pr. 8. cor. 4. — of ſo much the greater denſity.] This is not generally true, as appears by cor. 2 and 3.

[ib. cor. 3. — truely defined.] This is demonſtrated in Graveſand, L. IV. p. 232, 3. See Appendix, p. 20.

[Pr. 10. loſe almoſt a tenth part of its motion.] For by Schol. Pr. 40, p. 160, it is, $1 : 860 :: \frac{8}{3} : 2293.33 +$ = ſpace deſcribed in the time T; putting $D = 1$, $V = 1$; and $T : t :: 2293.33 : 2293.33 \frac{t}{T}$ = ſpace (uniformly) deſcribed in the time t. Then (pa. 161.) $\frac{t}{T} : 2,302$, &c. $\times$ Log. $\frac{T+t}{T} :: 2293.33 \frac{t}{T} : 229.5$; therefore $2293.33 \times 2.30258 \times \text{Log.} \frac{T+t}{T} = 229.5$; and $\log. \frac{T+t}{T} = ,043461$; and therefore $\frac{T+t}{T} = 1.105 +$; and $\frac{T}{T+t} = \frac{1}{1.105} = \frac{9}{10} +$ and $\frac{t}{T+t} = \frac{1}{10}$ nearly.

[ib. or as 75 (0^{12}) to 1 nearly.] In fig. 3. pl. 5. L. II. Let SA = rad. earth = 20949674 feet. AB = 1. AF = 200 miles = 1056000 feet; then, by cor. pr. 22. II. $Aa - Bb : Aa - Ff :: thin : thnz$; that is, $AB \times SF : AF \times SB ::$ or $1 : 1005325$

Fig. : : *thin* : *thnz* : : (by ſch. pr. 86 hyperbola) log. AH — log. BI : log. AH — log. FN; becauſe water is 860 times denſer than air, and the incumbent weight of the atmoſphere at the ſurface of the earth is about = weight of 33 feet of water; therefore the weights (and conſequently the denſities) in A and B, are as 28380 and 28379; whence AH or S*t* = 28380, and BI or S*n* = 28379; and log. AH — log. BI = ,0000152, log. AH = 4.4530124; therefore 1 : 1005325 : : ,0000152 : 4,4530124 — log. FN; and log. FN = — 11,1730. And therefore FN = ,000000000015, and denſity in F : denſity in A : : 15 : 28380000000000000 : : or as 1 : to 1800000000000000, which is leſs than 1 to 75000000000000; but the computation will vary as the radius of the earth, the denſity of water and air at the earth, is ſuppoſed to vary.

[ib. and hence the, &c.] as before (by Pr. 40. Sch.) ſince denſity of water : to denſity of air 200 miles from the earth : : 645 (0^{14}) : 1. Space deſcribed in time T = 172 (0^{15}). And $\frac{t}{T}$: log. $\frac{T+t}{T} \times 2.302$, &c. : : 172 $(0^{15}) \times \frac{t}{T}$ (= ſpace deſcribed in time *t*) : 2792250000 = ſpace Jupiter deſcribes in 1000000 years. Therefore log. $\frac{T+t}{T}$ = ,00000000705. And $\frac{T+t}{T}$ = 1.000000017. And $\frac{T}{T+t}$ = ,999999986. And $\frac{t}{T+t}$ = ,000000014, which is leſs than $\frac{1}{1000000}$.

[Pr. 13. are almoſt as 16, &c.] for (by this and cor. 2. Pr. 8.) accelerative force of Jupiter towards Saturn :

Saturn : accelerative force of Jupiter towards the Fig. Sun : : $25 \times \frac{1}{3021}$: 16×1 : : 81 : $\frac{16 \times 81 \times 3021}{25}$.

[Pr. 14. Sch. And hence we may find, &c.] For by cor. 7. prop. 66. I. The apſides of their orbits move in conſequentia. And by cor. 16, of the ſame, the motion of the apſides of the body P (plate 21. fig. 2.) will be as the periodical time of P directly, and the ſquare of the periodical time of T inverſely, that is, as the periodical time of P (becauſe the periodic time of T, round S, or of the Sun round Jupiter or Saturn, or (which is the ſame thing) of Jupiter or Saturn round the Sun, is given); that is (by prop. 15. I.) in the ſeſquiplicate ratio of their diſtances PT.

[Pr. 19. in the duplicate proportion of the rad. 50. to the coſine of latt.] For (by cor. 3. pr. 4. L. I.) centrifugal force at the equator : centrifugal force at *a* from the axis ($= ab$) : : rad. : coſ. lat. : : $r : c$. Alſo the force directly from the axis : force directly from the earth : : $ab : be$: : $r : c$. Therefore, *ex equo*, centrifugal force at the equator : centrifugal force directly from the earth, at a : : $rr : cc$.

[ib. but by computation (from] Let GC $= t =$ 51. 101. BP $=$ C $=$ 100. PE $= x$. Then will ED$^2 =$ $\frac{tt}{cc} \times \overline{cx - xx}$. And ER$^2 =$ PD$^2 = \frac{ttx}{c} - \frac{tt}{cc}x^2$ $+ x^2$. And ER $= \sqrt{\frac{tt}{c}x + \frac{cc - tt}{cc}x^2} =$ (ſuppoſe to) $\sqrt{r^2x - sx^2}$. Whence the fluxion of the area PQRE is $= \overline{r^2x - sx^2}|^{\frac{1}{2}}\dot{x} = rx^{\frac{1}{2}}\dot{x} - \frac{sx^{\frac{3}{2}}}{2r}\dot{x}$ $- \frac{s^2x^{\frac{5}{2}}}{2.4r^3}\dot{x} - \frac{3s^3x^{\frac{7}{2}}}{2.4.6r^5}\dot{x} - \frac{3.5s^4x^{\frac{9}{2}}}{2.4.6.8r^7}\dot{x}$ — &c. And the area PQRE $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{sx^{\frac{5}{2}}}{5r} - \frac{s^2x^{\frac{7}{2}}}{7.4r^3} - \frac{3s^3x^{\frac{9}{2}}}{9.4.6r^5}$

Fig. $-\frac{3.5.s^4x^{\frac{11}{2}}}{11.4.6.8r^7}$ — &c. $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{3.sx}{2.5r^2}A + \frac{5.sx}{7.4r^2}B + \frac{7.3.sx}{9.6r^2}C + \frac{9.5.sx}{11.8.r^2}D +$ &c. = (putting $\frac{sx}{rr} = q$) $\frac{2}{3}rx^{\frac{3}{2}} - \frac{3}{10}Aq + \frac{5}{28}Bq + \frac{7}{18}Cq + \frac{45}{88}Dq +$ &c. = (when $x = c = 100$; and s being $= \frac{cc - tt}{cc} =$,0201. And $r^2 = \frac{tt}{c} =$ 102,01. =) to 6693,39. And PBM = 5000. Whence PQRM = 1693.39. Therefore by cor. 2. pr. 91. I. force of the ſpheroid : force of the ſphere : : (when P and A coincide) $AS - \frac{AS \times PQRM}{CS^2} : \frac{AS}{3} :: 1 - \frac{PQRM}{CS^2} : \frac{1}{3} :: 1 - \frac{4 \times 1693.39}{10201} : \frac{1}{3} ::$ 1,008 : 1 : : 126 : 125.

[ib. p. 243. if the denſity, &c.] this will appear by conſidering what went before, and cor. 3. pr. 91. I.

[ib. but if the diurnal, &c.] for the (centripetal or) centrifugal force, is as the ſquare of the velocity when the rad. is given.

[ib. 244. — augmented in proportion as the force of gravity is diminiſhed.] for, reviewing the former calculation; if the denſity of the earth were greater than it is; the force of gravity, to the centrifugal force, would be greater than in that proportion of 289 to 1; (and the two diameters would be in a leſs proportion than 289 to 288; and alſo the diameters would be in a leſs proportion than 230 to 229;) conſequently the ratio of the difference of the diameters to either, decreaſes as the gravity increaſes. And on the contrary, that ratio increaſes as the gravity decreaſes. And upon this depends the foregoing proportion, as $\frac{4}{505} : \frac{1}{100} :: \frac{1}{289} : \frac{1}{229}$.

[ib. line 11.] read $\frac{29}{5} \times \frac{400}{94\frac{1}{2}} \times \frac{1}{229}$ to 1. In Fig. the Original, 2d edit. it is $\frac{29}{5} \times \frac{5}{1} \times \frac{1}{229}$ to 1, or as 1 to 8.

[Prop. 20. Whence ariſes this Theorem] for let 52. *nkyn* be a circle circumſcribed round the ellipſis *nbyl*. Then $kbm : fbc \ (=) \ abd :: kt^2 : fg^2$. And $ab = \frac{kbm \times fg^2}{kt^2 \times bd}$. But becauſe *kbm* and *kt* are given, and $bd =$ (nearly to) $2kt$ which is given. Therefore $ab \propto fg^2$; but the difference between the weight at *n* and *a*, is as $(at - bt =)\ ab$. Therefore the increaſe of weight is as fg^2, or the ſquare of the ſine of lat. at *a*, nearly, or (by trigonometry) as the verſ. of $2an$.

[*ib.* and the arcs of the degrees, &c.] The length of a degree is $\propto$ rad. of curvature, and in the points *n* and *b*, are (by ex. 1. pr. 19. Sect. II. Curve lines) as the parameters of *ny* and *bl*, that is, as bl^3 to ny^3, or 229^3 to 230^3, that is, as 56637 to 57382, as is inſerted in the Table, page 247.

But to find the rad. curvature at *b*; we have (by Ex. 2. Prob. 5. Sect. II. Fluxions) $mH = y + \frac{4 \times \overline{a-b}}{abb} y^3$; and therefore (putting *s* for the ſine of the angle *ntb*); rad. curvature *m*C at $b = \frac{y}{s} + \frac{4 \times \overline{a-b}}{abbs} y^3 =$ (becauſe $\frac{y}{s}$ is a given ratio,) $R + \frac{4 \times \overline{a-b}}{abb} Ryy$, very near, R being the radius of curvature in *n*. And therefore the increaſe of this radius, which is as the increaſe of the degrees, is as *yy* or *ss* nearly. And here $a-b$ being very ſmall, $\frac{4 \times \overline{a-b}}{abb} R$, is an extremely ſmall quantity.

Fig. [Pr. 23. — which I cannot here deſcend to explain.] Since the moon's orbit is more excentric than thoſe of Jupiter's ſatellites, the motion of the moon's apogee will be greater in proportion, than the motion of the apogee in any ſatellite.

53. [Pr. 27. The area, &c.] For the moment of the area is $\propto$ $cb \times ad$. But $ad \propto$ angle $bcd \times cb =$ horary motion $\times\ cb$. Therefore, area $\propto cb^2$ $\times$ horary motion.

[Pr. 28. But the attraction of the moon;] for let A = attraction of P towards T. Let the attraction or force LS be reſolved into the two, LM, SM; the firſt acting towards M the latter towards S. Then the whole attraction of P towards T is $A - ST + LM - TM$. Let $A - ST = F$. Then the attraction of P towards T in the ſyziges is $F - 2AT$; and in the quadratures $F + CT$. By prop. 25; F : ML : : 178725 : 1000. And the forces 2AT, and CT, will be to each other as 2000 AT, and 1000 CT, that is, as $\frac{2000}{CT \times N}$, and $\frac{1000}{AT \times N}$. And the force F in the ſyziges and quadratures will be as 178725 directly, and the ſquare of the diſtance from T reciprocally. Therefore, &c. (Note, $+\frac{2000}{CT \times N}$ is falſe printed for $-\frac{2000}{CT \times N}$, in line 10, pa. 268.)

[ib. — Quadrature in C; or, which —] for CTP : CTp : : angular motion of the moon from the ſun's quadrature : to its angular motion from the fixt point C, that is (becauſe the given circumference, or one revolution, is to be deſcribed in either caſe) : : periodic revolution : ſynodic revolution.

54. [ib. But by computation I find,] with the radii TA, TC, deſcribe the circles AIc, Cic. And let

AZ Cz, touch the ellipſis in A and C. And take AI to Ac, and Ci to Cc, as ∠ CTP to CTp. (fig. 5. pl. 10.) Fig. 54. Now in order to determine the curvature of the orbit, ſuppoſe the point a to coincide with A, and draw the parts of the orbit ARO, Cro, which will paſs between the ellipſis and circle; for TO or TE is leſs than TL, and greater than (TA or) Tc. And To (that is Te) is leſs than TC or Tc, and greater than Tl. Draw the lines TB, Tb, ſo that AB = Cb. And let the points A, I, C, and alſo C, i, c coincide; and the curvatures of the ellipſis, orbit, and circles at the points A, C, will be reſpectively as BE, BR, BI, and be, br, bi. And the difference of curvatures of the ellipſis and circles, and of the orbit and circles in A, C, will be as EI, RI, and ei, ri, reſpectively. But RI : EI or OC (becauſe TE = TO, and TI = Tc : :) $AI^2 : Ac^2$ (Lem. 11. I.) : : $CTP^2 : CTp^2$. Whence $RI = EI \times \frac{CTP^2}{CTp^2} = \overline{BI - BE} \times \frac{CTP^2}{CTp^2}$. Alſo, ri : oc or ei, or be — bi : : $Ci^2 : Cc^2$: : $CTP^2 : CTp^2$. Whence $ri = \overline{be - bi} \times \frac{CTP^2}{CTp^2}$. But in the ⊙ TA; BI, AI or AB, and 2AT are ∺. And in the ⊙ TC; bi, Ci or Cb, 2CT are ∺; whence $BI = \frac{AB^2}{2AT}$; and $bi = \frac{Cb^2}{2CT}$. Alſo in the ellipſis, $BE \times 2AT : BA^2 : : AT^2 : TC^2$. And $be \times 2CT : bC^2 : : CT^2 : AT^2$. Whence $BE = \frac{BA^2 \times AT}{2CT^2}$. And $be = \frac{bC^2 \times CT}{2AT^2}$. Now from hence, the curvature of the orbit at a : curvature of the orbit at C : : BR or BI — IR : br or bi + ir : : $\frac{AB^2}{2AT} - \overline{BI - BE}$

×

Fig.

$$\times \frac{CTP^2}{CTp^2} : \frac{Cb^2}{2CT^2} + \overline{be - bi} \times \frac{CTP^2}{CTp^2} :: \frac{AB^2}{2AT} \times$$

$$\frac{CTp^2}{CTP^2} - BI + BE : \frac{Cb^2}{2CT^2} \times \frac{CTp^2}{CTP^2} + be - bi ::$$

$$\frac{AB^2}{2AT} \times \frac{CTp^2}{CTP^2} - \frac{AB^2}{2AT} + \frac{BA^2 \times AT}{2TC^2} : \frac{Cb^2}{2CT^2} \times$$

$$\frac{CTp^2}{CTP^2} + \frac{bC^2 \times CT}{2AT^2} - \frac{Cb^2}{2CT} :: \frac{CTp^2}{AT \times CTP^2} -$$

$$\frac{1}{AT} + \frac{AT}{TC^2} : \frac{CTp^2}{CTP^2 \times CT^2} + \frac{CT}{AT^2} - \frac{1}{CT} ::$$

$$\frac{CTp^2 - CTP^2}{AT \times CTP^2} + \frac{AT}{TC^2} : \frac{CTp^2 - CTP^2}{CT \times CTP^2} + \frac{CT}{AT^2}$$

$$:: AT^3 + \frac{CTp^2 - CTP^2}{CTP^2} \times AT \times CT^2 : CT^3 +$$

$$\frac{CTp^2 - CTP^2}{CTP^2} \times CT \times AT^2.$$

55. [Pr. 29. The tangent of the angle CTP, &c.] On the axis DC deſcribe the circle C*d*D, and let the line T*b* revolve uniformly to *d*, in the ſame time that TP revolves to A, and deſcribes areas proportional to the times in the ellipſis CAD. Draw *be* ∥ to *d*T, and draw T*b*, TP. Since the circle and ellipſis are deſcribed in equal times, parts proportional to the whole will be deſcribed in any equal times; now area ellipſis : area circle : : TA : T*d* (= TC) : : *e*P : *eb* : : (*e*PC + *e*PT =) TPC : (*eb*C + *eb*T =) T*b*C. Therefore the point in the circle is at *b*, when the point in the ellipſis is at P. But TA : (T*d* =) TC : : *e*P . *eb* : : tangent $\angle$ *e*TP : tangent $\angle$ *e*T*b* (= $\angle$ of the mean motion.)

[ib. Square of the ſine of the angle CTP.] Theſe things are plain from prop. 26, where P*d* (fig. 4.) is the exceſs of the moment, and is as $\frac{PK^2}{PT}$.

56. [ib. which we may effect, &c.] Let A be the area deſcribed by the moon in any time, arch

CP = z, r = radius TC. PK = y, TK = u, Fig. 56.
t = tan. PTC. Then $\dot{A} = \frac{yy\dot{z}}{r} = y\dot{x}$, and A = Fl : $y\dot{x}$ = area CPK. But TPC expreſſes the mean motion, and therefore TPK is the equation, which is as TK × PK or uy.

But by the nature of the circle, $\dot{z} = \frac{rr\dot{t}}{rr+tt}$, and ſince $\dot{t}$ is in a given ratio to t, (or as .3123 to 69,) therefore $\dot{z}$ is as $\frac{rrt}{rr+tt}$ or $\frac{t}{\sqrt{rr+tt}} \times$ TK or $y \times$ TK, or as uy. Therefore decreaſing the tangent in the ſubduplicate ratio of 11073 to 10973, or in the ſimple ratio of 69 to 68, 6877; accelerates the area in proportion of PK^2, as it ought to do.

[ib.—in a proportion compounded of the duplicate, &c.] for (by cor. 16. pr. 66. L. I.) all angular errors are as the ſquare of the time of the moon's revolution, directly, and the ſquare of the time of the earth's revolution inverſly; that is (by pr. 15. L. I.) as the ſquare of the time of the moon's revolution directly, and the cube of the earth's diſtance from the ſun inverſly.

[Pr. 30. And this force by prop. 25, is, &c.] The force 3PK : force ML : : 3IT : PT (for theſe are the ſame) and force ML : centripetal force by which the moon revolves, &c. : : 1 : $178\frac{29}{40}$ (by pr. 25.) therefore *ex equo*, force 3PK : centripetal force the moon revolves with : : 3IT × 1 : PT × $178\frac{29}{40}$ Or as IT : rad. × 59,575.

[ib. the half of which the moon] it ſhould be, which the moon, by the action of the ſaid force, as it is in the firſt edition.

[ib. And the angle PTM is equal to the angle, 57.
&c.] for in this caſe LM is perpendicular to MP. Let PR be a tangent to the point P; then the triangle

Fig. angle RPT is a right one; and angle RPM = 57. angle PTM. And angle LPM : $\angle$ (RPM =) PTM (when the radius is PM) : : LM : RM : : force producing LM : force producing RM : : 1 : 59,575.

[Pr. 31. cor. and the decrement is to the remaining motion as 100, &c.] The motion of the nodes in the octants : motion in the ſyziges : : $11073^2 : 11023^2$: : (becauſe 11073, 11023, 10973 are in arithmetical, and nearly in ∺ progreſſion) 11073 : 10973; and decrement : remaining motion : : (11073 — 10973 =) 100 : 10973.

58. [ib. but the decrement] for the decrements are as the forces and times, or as the whole motions and times; that is, decrement at H (or increment at *b*) : decrement at A : : mot. at H, and time : mot. at A, and time : : (that is, by what went before) as mot. at H × $\overline{yy - \frac{1}{2}rr}$: mot. at A × $\overline{rr - \frac{1}{2}rr}$ (or mot. at A × $\frac{1}{2}rr$.) This reaſoning being very obſcure, you will find the motion of the nodes clearly inveſtigated, in (Prop. V. Sect. VI. and cors. of) my Aſtronomy.

[Prop. 32, it is drawn back again] that is, ſuppoſing the place of the node given.

[Prop. 32. Now the area of the ſemicircle] For let NT = 1, AZ = y, TZ = x. Then $eZ = \frac{y^3}{9.0827 + yy}$, and Ae $(AZ - eZ) = \frac{9.0827y}{10.0827 + yy}$.

But $yy = 1 - xx$, therefore $Ae = \frac{9.0827\sqrt{1 - xx}}{10.0827 - xx}$

$$= \frac{\overline{b-1}\sqrt{1-xx}}{b-xx} = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$$

&c. × into $\sqrt{1 - xx}$, putting $b = 10.08276$, and the fluxion area $NAe = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$, &

&c. $\times \dot{x}\sqrt{1 - xx}$; whoſe fluent (by form 17. Fluxions) $= \frac{1}{b} + \frac{1}{4bb} + \frac{1}{8b^3}$ &c. $\times \overline{b - 1}. \ \varphi$ (putting $\varphi =$ quadrant or $\frac{\mathrm{NA}n}{2}$) $= .92435\ \varphi$, therefore half the area $\mathrm{NF}n = .07565\varphi$. And the area φ (or quadrant) is to the area $\frac{\mathrm{NF}n}{2}$; or the ſemicircle, to the whole area $\mathrm{NF}n$; as 1 to .07565, or as 793 to 60.

Fig.

[Prop. 33. will nearly agree with] let N = 19 49 3 55, $p =$ periphery of NAn or DFB, E the equation, S = S.2NA.

Then NAZ repreſents the mean motion of the nodes for the arch NA, as N does for the whole circle; and NAZ is the true motion; and ATZ is the difference proportional to the equation. Divide by $\frac{1}{2}r$, and then $p : \frac{xy}{r} :: \mathrm{N} : \mathrm{E}$, or $p : \frac{2xy}{r} :: \frac{1}{2}\mathrm{N} : \mathrm{E}$, that is, $p : \mathrm{S} :: \frac{1}{2}\mathrm{N} : \mathrm{E}$; whence if BF = 2NA, then (by conſtruction) $\mathrm{AD} : \mathrm{CD} :: p : \frac{1}{2}\mathrm{N} :: \mathrm{S} : \mathrm{E}$. Suppoſe S an arch in DF, then $360 : \frac{1}{2}\mathrm{N} ::$ degrees in S : E; but deg. in S (in the circle DF) : degrees in DG (in the circle DG) :: AD : CD :: (by conſtruction) $360 : \frac{1}{2}\mathrm{N}$ in degrees :: degrees in S : E; whence the degrees in DG = E; or the angle DAG = equation.

[ib. cor.] for in the ſyziges the areas ANZ and eNZ vaniſh; and NA is 90°, when they are in the quadratures; and when in the octants, the ſine of BF = S.90 = 1; therefore,

38.3 — 1.58319
Rad. — 10.
1 — 0

S. < A (1° 30′) 8.41681

[Pr.

Fig. [Pr. 2. pa. 291, and by the demonſtration] the angle ATN will be the diſtance from the nodes true place, by conſtruction. Alſo the mean motion of the ſun, in the time NA : mean motion of the node from the ſun, in the ſame time : : (mean annual motion of the ſun : mean annual motion of the node from the ſun : : (by Pr. 1.) area of the ellipſis : area of the circle : :) TBN : TFN; but TBN was the mean motion of the ſun in the time NA; and therefore TFN is the mean motion of the node from the ſun in that time NA; and FTN the angle of the mean motion, or the diſtance of the ſun from the mean plane of the node.

59. [ib. cor.] for let $atn = ntb = 45°$, and $fc \perp$ to at; then fc is the ſine of fta, the Δ's fcb and btb are ſimilar; therefore $fc : fb :: bt : bb$; and by compoſition, $fb : ct$, or $at :: bf : fc$; and by inverſion fc is to at or $kt :: fb : fb$ or $by :: k\mathrm{H} : \mathrm{H}m$ or $tk + t\mathrm{H}$.

[ib. but the ſine of] draw $oz \perp$ to it, then oz is to fc in a ratio compounded of (*or* to fb or of) os to fg, and (the ſine of orz or trs to the ſine of fbc or tbg, that is, of) ts to tg; that is, in the compound ratio of $os \times ts$ to $fg \times tg$, that is (becauſe rad. : coſine : : 2^{ce} ſine : ſine of the double arch, and therefore the rectangle of the ſine and coſine being as the ſine of the double arch) as ſine of $2otn$, to ſine of $2ftn$ or radius.

[ib. Sch.] for TS : (TS + SK =) TK : : 360° : (360° + 39°,6355 =) 399,6355 : : 9,0827646 : 10,0827646. Therefore TH : TK : : $\sqrt{9{,}0827646}$: $\sqrt{10{,}08}$, &c. And therefore TH : HK : : 18,6524761 : 1 : : TS : SH : : mean motion of the ſun : mean motion of the node = 19° 18′ 1″ 23‴$\frac{2}{3}$.

Laſtly, TK + TH (= 38,22428 : KH (= 1) : : rad. : ſine of 1° : 29′ : 57″ (by cor.)

[Pr. 34. cor. 2. with due regard] for whilſt p moves from Q to F, the ſum of the areas is comprehended by TH produced, FQ, and tangent to Q; but in moving from F to *q*, the line H*p* falls on the other ſide of the circumference, and the ſum of theſe areas is that comprehended by FT produced, F*q*, and the tangent to *q*; and the difference of theſe areas is the ſemicircle, generated in the time of deſcribing QA*q*; and in the time of deſcribing the whole circumference, the whole circle will be generated. Fig.

[ib. cor. 4. that is, as the diameter] Suppoſe Q*p* to be the double diſtance of the moon from the quadratures; MK the ſine thereof. Then K*k* : M*p* : : MK : radius. And the ſum of all the K*k*'s, or the diameter : ſum of all the M*p*'s or QA*q* : : ſum of all the ſines : ſum of as many radii, or half as many diameters. And therefore (multiplying the antecedents by $\frac{Pp}{PG}$, and the conſequents by 2) the ſum of all the ſines $\times \frac{Pp}{PG}$: ſum of as many diameters : : diameter $\times \frac{Pp}{PG}$: whole circumference.

[Prop. 35. by the ſame increments as the ſine of inclination doth, by cor. 3.] For if AEG be double the diſtance of the nodes from the quadratures, GED will be double the diſtance of the nodes from the ſun; and they both have the ſame ſine.

[ib. Schol.] Here are ſeveral particulars about the moon's motion barely laid down; many of them being taken only from obſervations.

[ib. pa. 299. The force of this action is greater] It is greater the nearer it is, and attracts the moon from the earth, and ſo dilates the orbit the more. But the further off, the leſs force, and the leſs the moon is drawn from the earth. And at a greater diſtance

Fig. diſtance ſhe moves ſlower; and at a leſs diſtance, faſter.

[ib. further I found that the apogee] by cor. 14. prop. 16. B. I. And if a be the ſun's mean diſtance, and $a + x$ any other diſtance, then the motion will be as $\frac{1}{\overline{a+x}^3}$ or $\frac{1}{a^3} - \frac{3x}{a^4}$, or $\frac{1}{a^3} \times \overline{1 - \frac{3x}{a}}$, and the equations, as $\frac{x}{a}$, or as x.

The motion of the ſun, that is the angular motion, is reciprocally as the ſquare of the diſtance.

[ib. but if the motion,] let v be the velocity, m the mean motion of the ſun. Then if v be as $\frac{1}{\overline{a+x}^2}$ or $\frac{1}{aa} - \frac{2x}{a^3} = m - \frac{2x}{a}m$; then the equation is $\frac{2x}{a}m$. But if v be as $\frac{1}{\overline{a+x}^3}$ or $\frac{1}{a^3} - \frac{3x}{a^4} = m - \frac{3x}{a}m$, then the equation is $\frac{3x}{a}m$. And the firſt equation to the ſecond, is as $\frac{2x}{a}m$ to $\frac{3x}{a}m$, or as 2 to 3. And hence the greateſt equation of the apogee and nodes (ſum of all the $\frac{3x}{a}m$): their mean motion $(m) :: 2^\circ\ 54'\ 30''$: ſun's mean motion; the greateſt equations being as the mean motions.

[ib. pa. 300. By the theory of gravity I likewiſe found] For then the moon being nearer the ſun (in all points of its orbit taken together), the ſun's force muſt be greater.

[ib. p. 301. By the ſame theory of gravity the action,] For when the ſun is in the line of the nodes, its whole force is exerted in moving the moon, and none in moving the plane of its orbit.

[ib. By the ſame theory of gravity, the moon's] Fig. By cor. 8. prop. 66. B. I.

[Pr. 36, in other poſitions of the ſun,] by the reaſoning in cor. 19. pr. 66. L. I. depreſſion of the water at P : aſcent at P in the direction PH : : (LM : TM : :) PT : 3PK; and aſcent at P in direction PH : aſcent at P in direction TP : : (rad. : S.PTK : :) PT : PK; therefore (*ex equo*) depreſſion at P : perpendicular aſcent there : : ($PT^2 : 3PK^2$: : $\frac{1}{3}PT^2 : PK^2$; but the force PT that depreſſes P is given, therefore the force to raiſe the waters at P, is always as PK^2, that is, as the verſed ſine of 2PTC or twice the ſun's altitude.

Alſo the effects of theſe forces (by cor. 14. pr. 66. L. I.) at different diſtances from the ſun are reciprocally as the cubes of theſe diſtances; and therefore,

Cor. 1. The ſun raiſes the water under it to one Paris foot and 11 $\frac{1}{30}$ inches.

[Pr. 37, but becauſe of the reciprocation] ſee pr. 24.

[ib. and the ſun's force in—] let A be the poſition of the moon, and ATP = angle the ſun and moon makes at the earth's center; and let PT repreſent the force of the ſun, which divide into the forces PK, KT, PK acting in the direction of the moon increaſes her force, KT acting in a different direction, diminiſhes it; and therefore PK — KT (or the difference of the ſine and coſine of the angle ATP) is the abſolute increaſe of the moon's force; but this differs not much from the coſine of double the $<$ ATP, (in an angle of 18°$\frac{1}{2}$ the difference = ,6310189) and in the arches 0° 45° 90° they perfectly agree. Or thus, 60.

Let *qc* = moon's force, *ca* = ſun's force. If the 61. ſun is in *n*, let $< ace = 2 < acn$, then *qe* or *qb* = ſum

Fig. ſum of the forces, cb = ſun's force, in that poſition
62. = coſ. of $2 \angle acn$.

[ib. But further, the force of the moon] for let PT be the whole force of the luminary in the place P (or in D) the other ſide of (the earth or) its parallel; the force DT = the two forces DK, KT. When the luminary is in D, the force to move the water in the direction TP, is TK; and the difference of theſe, or (TP — TK =) PK is the difference of the forces in the points P, D; or the abſolute force to move the ſea at the lat. CTP. But PK is $\propto \square$ ſine of ATP, or as $\square$ coſ. CTP.

Then ſince L + S is the whole force when the moon is near the ſyziges, and L — S in the quadratures. Therefore 0,8570327 L muſt be ſubſtituted for L only in the quantity L — S, which repreſents her force in the quadratures.

[ib. from whence we have] alſo 1,017522 L muſt be ſubſtituted for L, in L + S, which repreſents her force in the ſyziges; and 0,9830427 L for L, in L — S, when ſhe is in the quadratures; for theſe repreſent her attractive forces in the ſyziges and quadratures.

[Pr. 38. as the accelerative gravity] for if the diameters be given, the forces to raiſe the water will be $\propto$ abſolute forces; and if the abſolute forces are given (the ſpheres will be reduced into ſimilar ſpheroids, for) the perturbating forces will be as the diameters (for the perturbating force is $\propto$ PT fig. pr. 25.) Therefore if neither be given, the forces to raiſe the water in the moon and earth, are in the compound ratio of the earth to the moon, and the moon's diameter to the earth's diameter; or as 1090 to 100; therefore the water riſes to $93\frac{3}{4}$ feet in the moon.

No notice is taken here, of the tides being leſs by reaſon of the moon's motion round the earth, than if ſhe ſtood ſtill in a given poſition. Since

the motion of the tides reaches 90° for every ebb- Fig.
bing and flowing; it would ſeem, that the ſaid motion could not be propagated thro' 90° to its greateſt height, in the time the moon ſtays in the meridian But (by a computation from pr. 44. II.) it appears, that water will oſcillate in a canal of 90° in length in $\frac{2}{3}$ of an hour; and therefore in that time the tides will be propagated to that diſtance, and riſe to their higheſt; but the poſition of the moon in that time is not ſenſibly altered; and therefore the tides would riſe no higher, tho' ſhe were always to ſtand in a given poſition. But this ſuppoſes that there is depth enough of ſea, to ſupply water ſufficient for the purpoſe.

[Lem. 1. to Pr. 39. to recede towards this ſide and that ſide] that is, all the parts of AC to act in a direction towards the ſun; the parts of CE, directly from it.

[Lem. 2. The matter in the circumference of every circle IK] by this is meant the circular ring (in the plane of the circle IK) comprehended between the ſurfaces of the ſphere and ſpheroid.

[Lem. 3. about the ſame axe] he means about its diameter.

[ib.] I. Let ABD be a ſpherical ſurface, and 63.
AEFD a circumſcribing cylindric ſurface, and their common thickneſs the infinitely ſmall line B*b* or A*a*. Divide AC into an infinite number of equal parts = *in* = $\dot{x}$, and erect the planes *id*, *ne*; then the ſurfaces of the ſphere and cylinder comprehended thereby are equal. Now, flux. mot. cyl. ſurface : flux. mot. ſph. ſurface : : (as their matter × velocity, and the matter is given, and therefore, as) CB × $\dot{x}$, or *id* × $\dot{x}$: *uf* × $\dot{x}$. And mot. cyl. ſurface : mot. ſph. ſurface : : as all the CB × $\dot{x}$: to all the *uf* × $\dot{x}$: : AD^2 : area of the circle ABDG.

Again. Let ABD be a ſphere, and AEFD its 64.
circumſcribing cylinder; ECA a cone, all revolv-

Fig. 64. ing round AD. Divide BC into an infinite number of equal parts $= bd = \dot{x}$. Thro' the points draw ſpherical ſurfaces *bgd* concentric to ABD, and cylindric ſurfaces *bf* terminated at the line EC. Then (by what went before) mot. cyl. ſurface *bdf* : mot. ſph. ſurface *bdg* : : AD^2 : circle ABDG. Therefore mot. all cyl. ſurfaces, or the ſolid EBC : mot. all the ſph. ſurfaces, or the ſphere : in the ſame given ratio of AD^2 : to circle ABDG.

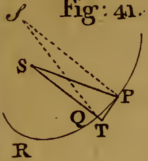
65. Further, flux. mot. cyl. round its axis $\propto$ flux. mot. × velocity, that is $\propto ef^2 \times \dot{x}$, or $x^2\dot{x}$. And the whole motion is $\propto \frac{x^3}{3}$. And therefore the motion of a cylinder round its axis $\propto$ length × cube of its diameter, × denſity; or as its weight × diameter.

66. Laſtly, let EBCA be a cylinder, and ECA a cone. Then flux. cyl. mot. : flux. con. mot. : : $AE^3 \times \dot{x} : ba^3 \times \dot{x} :: CA^3 \times \dot{x} : Cb^3 \times \dot{x}$, or as $r^3\dot{x} : x^3\dot{x}$. Mot. cyl. round its axis : mot. cone round its axis : : $r^3x : \frac{x^4}{4}$: : (when $x = r =$ CA) 4 : 1. Therefore, mot. cyl. : mot. fig. EBC (= difference of mot. of cylinder and cone) : : 4 : (4 — 1 =) 3. hence it follows,

Mot. cyl. : mot. ſol. EBC : : 4 : 3
Mot. ſol. EBC : mot. ſphere : : diameter □ : circle,
Therefore, *ex equo*,
Mot. cylinder : mot. of ſphere : : 4 diam. □ : 3 circle.
Or,
Mot. ſphere : mot. cylinder : : 3 circles : 4 diam. ſquare.

67. II. Flux. mot. cyl. : flux. mot. of a ring (or periphery) of the ſame diameter and matter : : (as velocity × flux. matter, as) $x^2\dot{x} : rx\dot{x}$. And mot. cyl. : mot. ring : : $\frac{x^3}{3} : \frac{rx^2}{2}$: : (when $x = r$) 2 : 3.

Fig: 41.



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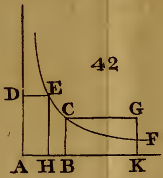

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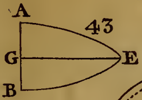

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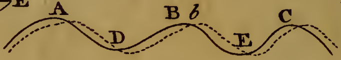

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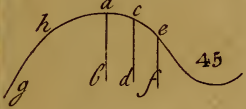

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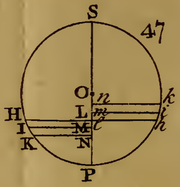

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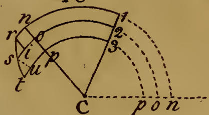

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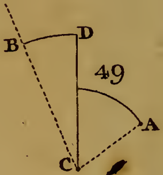

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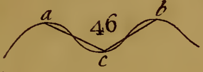

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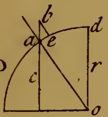

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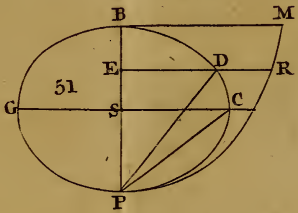

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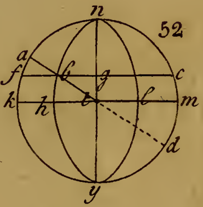

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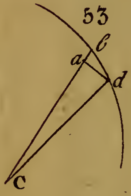

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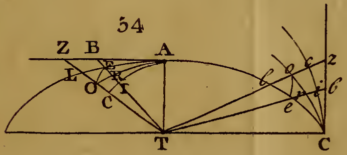

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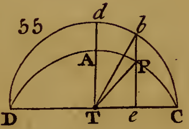

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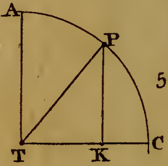

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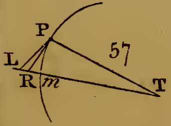

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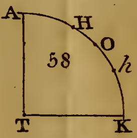

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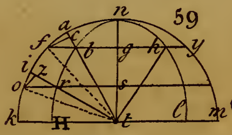

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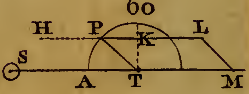

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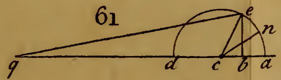

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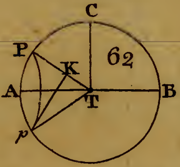

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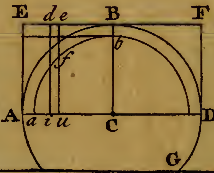


Principia

Pl: IV. pa: 98.

And mot. cyl. : mot. any ring : : 2 cylinders : 3 rings. Fig.

68. III. Let ED $= z$; and let $\dot{z}$ be given; then $y\dot{z} = r\dot{x}$. Now flux. mot. ring round C : flux. mot. ring round AB : : $r\dot{z} : y\dot{z}$ or $r\dot{x}$. Therefore mot. ring round C : mot. ring round AB : $rz : rx$: : (when $x = r$) AD : AC : : circumference : 2 diameters.

Therefore from the 1, 2, 3 Art. it follows, *ex equo*, that the motion of a ſphere : motion of a ring round their axes : : as the matter in the cylinder and ring, and 6 × circumference × circle : 24 diam. cube : : or as ſquare of the quadrantal arch : to ſquare of the diameter : : that is, as matter in the cylinder to matter in the ring, and 61685 : to 100000; or as the matter in the ſphere to the matter in the ring, and 925275, to 1000000.

[Pr. 39. (for the reaſons above explained),] for (by cor. 2. pr. 30.) mean horary motion of the nodes : $16'' \; 35''' \; 16'''' \; 36^{v}$: : $AZ^2 : AT^2$. And the ſum of all the mean horary motions throughout the year : ſum of as many $16'' \; 35'''$, &c. : : ſum of all the AZ^2 : ſum of as many AT^2 : : (by Lem. 1. pr. 39) 1 : 2; for the diurnal rotation, and annual motion of the earth, are ſuppoſed uniform.

[ib. But becauſe the plane of the equator.] For by prop. 30, the diſturbing force is as 3PK, (and in that prop the inclination of the planes being ſmall is not taken notice of,) but when the plane of the orbit of P is inclined to the ecliptic, the line PK is projected into *p*K, and the force will therefore be diminiſhed, in the ratio of PK to *p*K, or as the radius to the coſine of inclination; for the difference of the diſtances of P and T from the ſun become leſs in that proportion, and the difference of the forces are diminiſhed in the ſame ratio.

This being a propoſition of great difficulty, depending upon very intricate calculations, which few people are judges of; for this reaſon, the Author's demonſtrations have been objected to, and cenſured by ſome people as not true. And Mr. Simpſon (in his Miſcellaneous Tracts) mentioning ſome of theſe Cavillers, falls into the ſame notion. He has invented a ſort of motion (which he calls *Momentum*) unknown to Sir I. Newton, or any body elſe, and which differs from Sir Iſaac's, in the ratio of 800000 to 925725. But I never before heard of any motion that was not made up of the quantity of matter and velocity.

Another frivolous objection he makes, is about the motion of a ring being different from that of the equator. And he tells us, that the motion (*Momentum*) of a ring round its diameter is only half of what it would be, when revolving in its plane round the center. But it is more than half, for it is as 1 to $\frac{3.1416}{2}$, as is demonſtrated in Art. III. What he writes afterwards (tho' he ſays; *it is evident*,) is not intelligible. But he concludes at laſt in his way, that Sir I. Newton has made the preceſſion (by the ſun's force) to be but half of what it ſhould be. And as ſome he mentions had made the whole propoſition erroneous, he modeſtly aſcribes but *two miſtakes* to Sir Iſaac in this one propoſition. But I believe, that whoever reads the foregoing Notes, will ſoon be convinced, that his demonſtrations are all right, and that all theſe blunders they tell us of, are entirely of their own making, and muſt be aſcribed to themſelves only.

[Lem. 4. p. 326. Wherefore if both the quantity of light,] for quantity of light is as the quantity the body receives from the ſun directly, and the ſquare of the diſtance from the body reciprocally;

cally; that is, $\propto$ ſquare of the apparent diameter directly, and ſquare of the body's diſtance from us, reciprocally: therefore the body's diſtance is as the apparent diameter directly, and $\sqrt{\text{light}}$ reciprocally. Fig.

[Pr. 40. cor. 4.] For then the parabola and earth's orbit touch in the vertex, and the area there will be $= \frac{1}{2}$ velocity × diſtance or radius or $\frac{1}{4}$ parameter, that is $= \frac{10000 \times 243.2747}{2}$, and $\frac{10000 \times 10.1364\frac{1}{2}}{2}$ for the diurnal and horary motion. The reſt is plain from Pr. 14. L. I.

[Lem. 5.] For the ordinates AH, BI, CK, &c. Put a, $2a$, $3a$, &c. and take the differences as follows. 69.

$$\begin{array}{ccccccccccc}
a & & 2a & & 3a & & 4a & & 5a & & 6a \\
 & b & & 2b & & 3b & & 4b & & 5b & \\
 & & c & & 2c & & 3c & & 4c & & \\
 & & & d & & 2d & & 3d & & & \\
 & & & & e & & 2e & & & & \\
 & & & & & f & & & & &
\end{array}$$

Now, to find any ordinate as $4a$, we have

$$4a = 3a - 3b = \begin{array}{l} 2a - 2b \\ -2b + 2c \end{array} = \begin{array}{l} a - b \\ -2\times b + 2\times c \\ \qquad\quad + c \end{array}$$

$$+ d = a - 3.b + 3.c - d.$$

and ſo of others.

Fig. Caſe I. If we ſubſtitute for p, q, r, &c. their
69. equals, we ſhall have

$$\begin{aligned} RS = {} & a \\ & + b \times -SH \\ & + c \times -SH \times -\frac{IS}{2} \\ & + d \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \\ & + e \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \\ & + f \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \times \frac{SM}{5} \text{ \&c.} \end{aligned}$$

Now let RS fall upon any ordinate, as for example on LD, then SH, SI, SK, SL, &c. will be $=$ LH, LI, LK, o, &c. $= 3, 2, 1, o$, &c. reſpectively, which being written in the aforeſaid ſeries, we have

$$RS = a - 3.b - 3 \times \frac{-2}{2} c - 3 \times \frac{-2}{2} \times \frac{-1d}{3}$$

$+\, o\, e = a - 3.b + 3.c - d$, which, by what went before, is the value of the ordinate DL; and ſince this holds generally, it is plain this ſeries will give the value of RS wherever it falls.

Caſe II. Let x repreſent any baſe HS, y any ordinate RS. Aſſume $y = A + Bx + Cx \times \overline{x - P} + Dx \times \overline{x - P}, \times \overline{x - Q}$, continued to as many terms as there are ordinates, which ſuppoſe four; P, Q, R, &c. being reſpectively $=$ HI, HK, HL. Then putting AH, BI, CK, DL, ſucceſſively for y, and o, HI, HK, HL for x, we ſhall have from the general equation, $AH = A$, $BI = A + BP$, $CK = A + BQ + CQ \times \overline{Q - P}$, $DL = A + BR + CR \times \overline{R - P} + DR \times \overline{R - P} \times \overline{R - Q}$, that is, from the figure,

AH

Fig. 69.

$AH = A = a,$
$BI = A + B \times HI.$
$CK = A + B \times HK + C \times HK \times IK.$
$DL = A + B \times HL + C \times HL \times IL + D \times HL \times IL \times KL.$

Whence by Subſtraction.

$AH - BI = - B \times HI,$
$BI - CK = - B \times IK - C \times HK \times IK.$
$CK - DL = - B \times KL - C \times \overline{HK + IL} \times KL - D \times HL \times IL \times KL.$

Then dividing by the coefficients of B.

$$\frac{AH - BI}{HI} = - B = b.$$
$$\frac{BI - CK}{IK} = - B - C \times HK = 2b.$$
$$\frac{CK - DL}{KL} = - B - C \times \overline{HK + IL} - D \times HL \times IL = 3b.$$

Again by Subtraction.

$b - 2b = C \times HK.$
$2b - 3b = C \times IL + D \times HL \times IL.$ And dividing
$$\frac{b - 2b}{HK} = C = c.$$
$$\frac{2b - 3b}{IL} = C + D \times HL = 2c.$$ Alſo
$c - 2c = - D \times HL,$ and
$$\frac{c - 2c}{HL} = - D = d.$$

Thus the coefficients A, B, C, &c. are determined for four ordinates, and they are found the ſame way, if more ordinates are given. Then taking x at pleaſure, as ſuppoſe $= HS$, then RS or y will be $a + bx + cx \times \overline{x - P} + dx \times \overline{x - P}$

Fig. $\times \overline{x - Q}$, &c. $= a + b \times p + c \times p \times IS + d \times p \times IS \times SK$, &c. $= a + bp + cq + dr$, &c.

69. [Cor. Lem. 8.] for then $AC : A\delta (= AI - \delta I) :: A\mu C :]\ A\delta dy - d\mu x (= AI\mu y - \delta I\mu X)$, and $:: ASC : AS\delta$ (Propor. 10. $:: (A\mu C + ASC =) ASC\mu A : (A\delta dy - d\mu x + AS\delta = AS\delta dyA - d\mu x =$ (becauſe Δ's $x\delta\mu$ and $\delta S\mu$ are equal) $AS\delta dyA - d\mu x + x\delta\mu - S\delta\mu = AS\delta\mu dyA - S\delta\mu =) AS\mu dyA ::$ time of deſcribing $A\mu C$: time of deſcribing $Ay\mu$.

[ib. Schol.] For Bn will cut the chord AC in a point a little beyond E (towards C), ſuppoſe at e. Now the area $AEX\mu A$ to the area $ACY\mu A$, is in ſomething a greater ratio than AE to AC, as ſuppoſe as Ae to AC, more near than before. Therefore $ASEX\mu A : ASCY\mu A :: Ae : AC$. Whence $ASBX\mu A$ (or $ASEX\mu A$) $: ASCY\mu A :: Ae : AC$. Or $ASBY\mu A : ASCY\mu A :: (Ae : AC ::)$ time in AB : time in AC, very near.

[Lem. 10. to the triangle ASC, that is,] $AC\mu A : ASC :: AC \times \frac{2}{3}M\mu : AC \times \frac{1}{2}SM$ (becauſe the <'s SMA and $AI\mu$ are equal, and $M\mu = I\mu$.) $:: AC \times 1\frac{1}{3}M\mu : AC \times SM :: MN : SM$. And by compoſition $ASC\rho A : ASC :: SN : SM$.

[Lem. 11. Subduplicate proportion of 1 to 2,] and therefore $= \frac{AC}{\sqrt{2}}$. And the arch in half the time $= \frac{AI}{\sqrt{2}}$. And $\frac{AI^2}{2} \div 2SP = \frac{AI^2}{4SP}$ = ſpace deſcribed in deſcending.

[Prop 41. and BE, by Lem. 11, is a portion,] tS and this hypothenuſe (mentioned in the prop.) are the diſtances of the earth and comet from the ſun. Now BE to tV is compounded of BE to the part of the hypothenuſe (projected into BE) or of BS to the whole hypothenuſe, and of that part of the hypothenuſe to tV, that is, as the gravitating forces at the comet and earth (for theſe lines would Fig. be deſcribed by falling bodies in equal times, by theſe forces), that is, as St^2 to hypothenuſe ſquare. And therefore $BE : tV :: BS \times St^2 :$ hypothenuſe cube.

Further, becauſe of the immenſe diſtance of S, 70. any of the points B, i, μ, are nearly in the curve of the parabola, and μ the vertex, and μp, $i\lambda$, $q\sigma$ nearly parallel. Therefore $< I\mu i =$ (by reaſon of the rectangle $Ii\mu\lambda$) $< p\mu L$ or $= S\mu L$. Therefore μI is the diameter to the vertex μ; alſo $\xi\sigma =$ (by conſtruction) $3S\sigma + 3i\lambda = 3S\sigma + 3I\mu = 3S\sigma + 3\mu p =$ (becauſe $2O\sigma = 3\mu p$) $3S\sigma + 2O\sigma$. And therefore $\xi O = 3S\sigma + 3O\sigma = 3OS$. But $\mu O : I\mu :: p\sigma : Ip$. But μI is biſected by $i\lambda$; therefore $Ip = 2I\lambda$; and $p\sigma = I\lambda = \frac{1}{2}Ip$ (ſince $I\sigma = 3I\lambda$ by conſtruction); therefore $\mu O = \frac{1}{2}\mu I$. Whence (by Lem. 8.) a line drawn from ξ divides the chord AC nearly as the time; whence BE drawn towards ξ is rightly drawn. But (by conſtruction) new BE : former $BE :: BS^2 : \overline{S\mu + \frac{1}{3}i\lambda}|^2 ::$ as the gravitating force at the diſtance $S\mu + \frac{1}{3}i\lambda$: to gravitating force at the diſtance B or μ :: ſpace fallen thro' at the diſtance $S\mu + \frac{1}{3}i\lambda$, in the time of deſcribing the arch $\frac{1}{2}A\mu C$: ſpace fallen thro' at B or μ, in the ſame time. But former BE = ſpace fallen thro' at μ. Therefore new BE = ſpace fallen thro' at the diſtance $S\mu + \frac{1}{3}i\lambda$, (in half the time the comet deſcribes the arch intercepted between TA, $\tau C =$ (by Lem. 11.) $i\lambda$, or this new BE nearly; which therefore is rightly determined, nearer than before.

Becauſe the diſtance of the ſun and comet is ſomething more than the hypothenuſe of the triangle, whoſe baſe is $S\mu$ and perpendicular IO (becauſe O is only a point in the chord, which ſhould be in the arch), therefore he ſuppoſes that diſtance = hypothenuſe (whoſe baſe is $S\mu + \frac{2}{3}i\lambda$, and perpendicular

Fig. 70. pendicular IO, that is) = DO. Alſo MP : X : : (by conſtruction) $\sqrt{St} : \sqrt{OD}$: : (by cor. 6. pr. 16. I.) velocity of the comet at diſtance DO : velocity thereof at diſtance S*t* : : ſpace deſcribed (uniformly) at the diſtance DO, and in the time of deſcribing the arch ABC : ſpace deſcribed (uniformly) at the diſtance S*t*, in the ſame time, which is = X by conſtruction; and therefore MP = ſpace deſcribed (uniformly) at the diſtance DO, (in the time that the arch ABC is deſcribed) = (by Lem. 10. cor.) to the chord of that arch.

The points *e*, *a*, *c*, *g*; and ε, α, χ, γ, being found out as E, A, C, G, before; is with an intent to find at laſt MP = MN, or AC = AG. Wherefore ſeveral of them being thus found, and a ſimple curve (viz. a circle) drawn thro' theſe points, finds the points Z and X where thoſe lines would be equal; for the nearer they approximate to equality, the nearer they come to the true points of the comet's orbit.

Further, velocity of a comet at Q (in a parabola) : velocity of a comet at Q (in a circle) by cor. 7. pr. 16. I.) : : $\sqrt{2} : 1$: : (nearly as) $\frac{4}{3} : 1$. Alſo velocity at Q (in a circle) : velocity at *t* (in a circle, by cor. 6. pr. 4. I) : : $\sqrt{St} : \sqrt{SQ}$. Therefore, *ex equo*, velocity of the comet at Q, in a parabola : velocity of the earth in its orbit at *t* : : $\frac{4}{3}\sqrt{St} : \sqrt{SQ}$. But velocity of the comet at Q : velocity of the earth at *t* : : (nearly as) AC : Tτ. Therefore AC : Tτ : : $\frac{4}{3}\sqrt{St} : \sqrt{SQ}$; or AC : $\frac{4}{3}$Tτ : : $\sqrt{St} : \sqrt{SQ}$. As it is by conſtruction (for AC is to $\frac{4}{3}$Tτ in the reciprocal ſubduplicate ratio of SQ to S*t*, and not in the direct ſubduplicate ratio, as is falſely printed; therefore Q is (nearly) in the chord of the parabola, and B a point of the comet's orbit, nearly.

Laſtly, if MP = MN, or AG = AC. Then Y*b* : YB : : Y*c* : YE : : *ac* : AC : : velocity in *b* :

velocity in B : : $\sqrt{SB} : \sqrt{Sb}$. But if $Sb = SB$ Fig. 70. and MP or AG invariable; it will be $Yb : YB$: : as ac or $AG : AC$, when the point G falls in CY. Therefore univerſally $Yb : YB : : AG \times \sqrt{SB} : AC \times \sqrt{Sb} : : MP \times \sqrt{SB} : MN \times \sqrt{Sb}$, to find the point b true.

[ib. p. 352. may be ſeen in the following table.] The places of the comet in this and all the other tables, are the geocentric longitudes and latitudes of the comet.

[ib. p. 358. And thinking, &c.] In plate 18, while the comet paſſes thro' I, K, L, M, &c. the earth paſſes from between P and K towards G. Whence the comet will move ſwift thro' LKL, and NOP; ſlower thro' LBM, and QRS, as is ſaid in the foregoing page 357.

[ib. p. 360, as about 6 to 1000,] for by page 348,9. $\frac{\text{lat. rect.}}{4} = 6$, nearly; when earth's diſt. $= 1000$.

[ib. p. 366. I found, that at the height, &c.] 71. Let SA = rad. earth = 20950000 Engliſh feet, nearly, AB = 850, AF = AS, and the denſities AH and BI are as 33 to 32. But (by cor. 2. p. 22. II.) $Aa - Bb : Aa - Ff : : thin : thnz$; that is, $AB \times SF$, or $2AB \times SA : AF \times SB$, or $SB \times SA : : thin : thnz$; or $2AB : SB : : thin : thnz$; that is, $1 : 12324 : : thin : thnz : :$ (by ſchol. prop. 86. hyperbola,) log. St — log. Su : log. St — log. Sz : : ,013364 : 1.518514 — log. Sz. Whence log. $Sz = -164.820578$; and SZ or FN =, (0^{163} cyphers) 66 +. Wherefore FN : AH : : ,0^{163} 66 : 33 : : or as $2 : 10^{165} : : 1 : 10^{164}$: : denſity in F : denſity in A. But diameter of Saturn's orb is = 191 times the ſemidiameter of the orb's magnus = 191×20000 radii of the earth $= 191 \times 20000 \times 20950000 \times 12$ inches $= 96\,0^{13} +$ inches. And a ſphere of 1 inch diameter : to this ſphere : : as 1 :

Fig. $\overline{960^{13}}|^{3} :: 1 : 88\,0^{43} +$; but $10^{164} > 8\,0^{44}$. Therefore, &c.

[Prop. 42.] the correction of the orbit here given is computed by the rule of falſe; 1ſt, for the longitude of the node, by operation 1 and 2, and 2dly for the inclination of the orbit, by operation 1 and 3.

I. for the longitude of the node; ſince (by conſtruction) time of deſcribing D : time of deſcribing E : : D : E : : G : 1; and (time of deſcribing E, nearly =) time B : time A : : 1 : C; therefore (*ex equo*) time of deſcribing D : time A : : G : C; therefore G — C is the error of the times between the firſt and ſecond obſervations, for the given longitude of the node K, and the given inclination of the orbit I. Again,

Time of deſcribing d : time of deſcribing e : : $d : e :: g : 1$; and time of deſcribing e (nearly =) time B : A : : 1 : C; therefore time of deſcribing d : time A : : g : C; and g — C is the error of time, between the two firſt obſervations, for the longitude K + P, and inclination I; therefore by the rule of falſe, by two poſitions (which ſays, diff. er. : diff. poſitions : : either er. : correction of its poſition), $G - g : G - C :: P : \frac{G - C}{G - g} P =$ the correction of K, or the computation may be thus, ſince to the ſuppoſition K and K + P, the errors between the firſt and third obſervations, in theſe two caſes, are T — S, and t — S; therefore $T - t : T - S :: P : \frac{T - S}{T - t} P =$ the ſame correction of K; therefore mP (is to be) $= \frac{T - S}{T - t} P = \frac{G - C}{G - g} P$; and therefore $mT - mt = T - S$; and $mG - mg = G - C$.

II. For the inclination of the orbit, it is plain Fig. $T - S$ = firſt error of the times between the firſt and third obſervations, to the given node K, and inclination I; and $r - S$ = error between the firſt and third obſervation, to the node K, and inclination $I + Q$. Therefore (by the rule of falſe), $T - \tau : T - S :: Q : \frac{T - S}{T - \tau} Q$ = correction of I; again this time of deſcribing δ : time of deſcribing $\varepsilon :: \delta : \varepsilon :: \gamma : 1$; and (time of deſcribing ε = nearly) time $B : A :: 1 : C$: *ergo* time of deſcribing $\delta : A :: \gamma : C$; therefore $\gamma - C$ = error of the times between the firſt and ſecond obſervations, to the node K and inclination $I + Q$; and $G - C$ was the error between the firſt and ſecond obſervations, to the node K and inclination I; therefore (as before) $G - \gamma : G - C :: Q : \frac{G - C}{G - \gamma} Q$ = correction of I; therefore nQ (muſt be) $= \frac{T - S}{T - \tau} Q = \frac{G - C}{G - \gamma} Q$; and $nT - n\tau = T - S$, and $nG - n\gamma = G - C$. Laſtly, from I and II.) $2T - 2S = mT - mt + nT - n\tau$; and $2G - 2C = mG - mg + nG - n\gamma$; therefore the numbers m, n are rightly found.

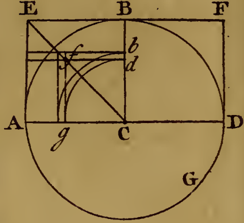
[ib and laſtly, if in—] becauſe the longitudes in the firſt and ſecond orbits are K and $K + mP$, therefore the latus rectums will be R and $R + \overline{r - R} \times m$; and becauſe in the firſt and third orbits or planes thereof, the inclinations are I and $I + nQ$; therefore the latus rectums will be R and $R + \overline{\varrho - R} \times n$; or $R + \overline{r - R} \times m$, and $R + \overline{r - R} \times m + \overline{\varrho - R} \times n$.

Or thus; if the longitude K be increaſed (in op. 1, 2.) by P, then (becauſe the diſtance of the perihelion from the ſun is increaſed accordingly), the latus rectum will be increaſed by $r - R$, and if the longitude be increaſed by mP, the latus rectum will be increaſed by $m\overline{r - R}$. And for the ſame reaſon (in op. 1, 3.) if the inclination be increaſed to nQ, the latus rectum will be increaſed to $N\overline{\varrho - R}$ (for at the increaſe Q, the increaſe of the latus rectum is $= \varrho - R$, ſince the whole is ϱ, or $R + \overline{\varrho - R}$); therefore if both longitude and inclination be increaſed, the latus rectum will be increaſed in both theſe reſpects, and will become $R + mr - mR + n\varrho - nR$.

Alſo when K becomes $K + P$, the tranſverſe becomes $\frac{1}{L + \overline{l - L}}$ or $\frac{1}{l}$; alſo when K becomes $K + mP$ the tranſverſe becomes $\frac{1}{L + m.\overline{l - L}}$, for the ſame reaſon when I is increaſed to $I + nQ$, the tranſverſe is $\frac{1}{L + n.\overline{\lambda - L}}$. Therefore when both longitude and inclination are increaſed, the tranſverſe will then be $\frac{1}{L + ml - mL + n\lambda - nL}$.

[ib. Sch. pa. 385, the comet] for by pa. 360, comet's diſt.: earth diſt. from $\odot$ :: 6 : 1000 :: diſt. comet in $\odot$ diameters : (earth's diſt. in $\odot$ diam. =) 109 $\odot$ diam. therefore comet's diſt. from the ſun's center $= \frac{654}{1000} = 654$ diams. and from its ſurface $=,154$ diams. $= \frac{1}{6}$ ſun's diameter.

Fig: 64



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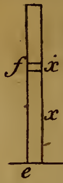

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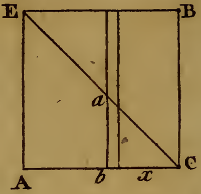

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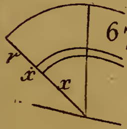

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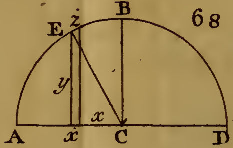

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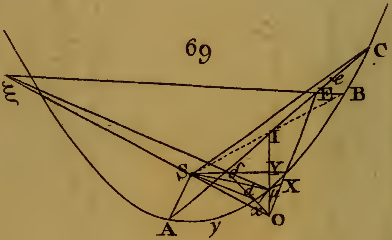

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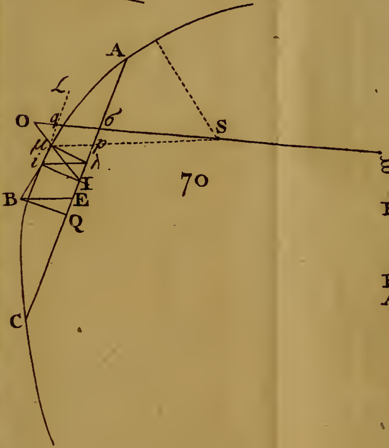

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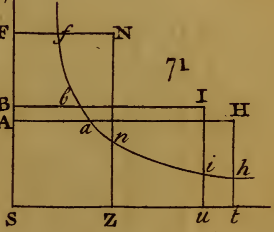

A DEFENCE OF

Sir I. NEWTON,

AGAINST THE

Objections that have been made againſt ſeveral Parts of the Principia.

Raro antecedentem ſceleſtum,
Deſeruit pede pœna claudo. Hor. Lib. III.

THIS incomparable Treatiſe being written in a conciſe ſtile, and in the ſynthetic method, and being upon ſubjects quite new and untouched before; the generality of readers could make little of it. As it contained a new ſyſtem of Philoſophy, built upon the moſt ſublime Geometry, the greateſt mathematicians were obliged to ſtudy it with great care and attention; and few became maſters of the ſubject; ſo that for a long time it was little read. But at laſt, when the value of it became more known, it gained univerſal approbation; and the whole world ſtood amazed at the numberleſs new diſcoveries contained therein. And upon account of its univerſal agreement with all the phænomena of nature; it was adopted as the only true ſyſtem by all, except ſome few that, thro' envy or ignorance, were bigotted to ſome other ſcheme.

It is true, many things therein depend upon very difficult mathematical computations, not eaſily comprehended by every reader; and this has given occaſion to ſome people to queſtion or even deny the truth of ſeveral propoſitions laid down in this book; and have therefore given computations of their own, different from his, which they aſſert for truth; and thereby, as they think, proving his to be falſe. I ſhall therefore ſpend a little time in anſwering the moſt material of theſe objections, and ſhall ſhew that theſe principles objected againſt, are all true in the ſenſe Sir Iſaac meant them; and that theſe objections are only the effect of their own ignorance or inattention.

J. Bernoulli (Tom. IV. pa. 340) thinks Newton's demonſtration, of cor. 4th to the laws, is imperfect, where he ſays, *only the action of two bodies is conſidered.* And lays down a method, by which he thinks it may be fully demonſtrated. But at laſt leaves it undemonſtrated himſelf; and tells you, that this is the foundation, but leaves the calculation for others to make out.

But Sir I. Newton, after he had ſhewn the truth of the cor. in two bodies, ſhews that the common center of gravity of theſe two bodies and a third, will either be at reſt, or move uniformly in a right line. And from thence, that the center of gravity of *any two* (in a ſyſtem of bodies) ſuffers no change of ſtate by their actions upon one another. From whence it follows, that in the whole ſyſtem, all theſe actions (compoſed of the action of *every two*) can induce no change. And if the actions of *any two* makes no difference in the center of gravity of the whole, ſeparately conſidered; then the actions of *every two* conſidered collectively can make no difference. This is certainly ſo, tho' delivered in a few words, Sir Iſaac was not writing to children.

The ſame *Bernoulli* (pa. 347) objects againſt the 10th prop. B. II. for not being ſolved in a way general enough to pleaſe him. Here Sir Iſaac has ſolved the problem according to that law of reſiſtance which really exiſts; but could not ſpare time, or fill up his book with uſeleſs enquiries, or things of no conſequence. His noble ſtructure the *Principia* is all gold, but he has left the droſs for ſuch authors as this, who are fond of any thing, and will ſerve well enough for building up their ſyſtems. This author in particular is always depreciating Sir Iſaac, and extolling his own performances; tho' they are long, tedious and laborious to the laſt degree, and often falſe; altho' he has the advantage of the analitical and fluxional methods; and ſeems to be ignorant that the method of analiſis, by which he (*Bernoulli*) ſolves theſe prolems, is incomparably ſhorter than the method of compoſition, in which Sir Iſaac has writ. Had he written the Principia in the analitic way, or according to the method of Fluxions, a ſcience then utterly unknown to the world; it could not have been read by any man living. By the ſame way of reaſoning this boaſting mathematical bully may as well condemn *Euclid*'s Elements, or *Apollonius*' Conics; becauſe moſt of the propoſitions may be demonſtrated incomparably ſhorter by common Algebra.

This Author ſeems to have a particular ſpite againſt the Engliſh mathematicians, being always carping and criticiſing. His works are full of invectives againſt Sir I. Newton for his great diſcoveries, which this low Critic is unſucceſsfully endeavouring to imitate. When one perſon has another man's works to look thro', he muſt be a great blunderbuſs that cannot make ſome ſmall additions; a man placed upon another's ſhoulders, will ſee further than his ſupporter. Yet in many caſes he has diſimproved Sir I. Newton. Sir Iſaac had a work entirely new to execute, in a ſhort, and general way; and could not take notice of every bauble. This author has had nothing to do but to imitate. The regular works of the one will live thro' all ages; but the confuſed chaos of the other will ſink into everlaſting oblivion.

The ſame author (pa. 484) alſo objects againſt the demonſtration of the 36th prop. B. II. For ſays he, *ſince in Newton's cataract there is no compreſſion of the water in any place, nor againſt the ſides of the cataract, the external water preſſing inwards, muſt diſturb the cataract, and mix with it. And therefore the Newtonian explication being contrary to the laws of Hydroſtatics cannot ſubſiſt.* Yet what Newton ſays (Caſe I.) might have ſatisfied him. His words are; *For let the ice in the veſſel diſſolve into water, yet will the efflux of the water, as to its velocity, remain the ſame as before. It will not be leſs, becauſe the ice, now diſſolved, will endeavour to deſcend. It will not be greater, becauſe the ice now become water, cannot deſcend without hindering the deſcent of other water, equal to its own deſcent.* But this author is eternally cavilling at every thing; and all his whole ſection on this ſubject, conſiſting of about 100 pages, where he has toſt this prob. about into all forms, is nothing but a heap of abſurd inconſiſtent ſtuff, neither agreeable to theory nor experiment. So we ſhall leave him here drowned in a *gurges* of his own contriving.

In the demonſtration of prop. 47, B. II. Sir I. Newton aſſumes for a principle, that the parts of the air hath a motion excited in it, by ſome cauſe or other, according to the laws of an oſcillating pendulum. But *Euler* has found fault with him for making ſuch an hypotheſis; and has himſelf aſſumed ſome different hypotheſes, from which he has pretended to demonſtrate, that the motion of the particles of air, are moved according to ſuch hypotheſes; and that his demonſtration for his hypotheſis, is as valid as Newton's is for his. But Sir I. Newton's hypotheſis is more than an aſſumed one, for it is true in fact. For any tremulous or vibrating body, from which ſound is propagated, is known to vibrate according to this law, aſſumed by Newton; and the like motion is communicated to the particles of air. Therefore he has rightly aſſumed, that the medium has ſuch a motion excited in it; and conſequently that it will continue to move according to that law. But there is no ſuch law of a vibrating body as *Euler* aſſumes, and therefore his demonſtrations come to nothing. To find the velocity of ſound is a very ſubtle problem; and they that would ſee it truly reſolved, may find it in my book of Fluxions, prob. 23, Sect. III.

In B. II. prop. 52, Sir I. Newton has inveſtigated the properties of vortices, in order to ſee whether the celeſtial motions can be accounted for thereby. And he has ſhewn that the celeſtial bodies are not carried round in vortices. But *I. Bernoulli*, a perſon of eternal contradiction, has undertaken to defend the notion of vortexes; tho' ſhewn to be abſurd and impoſſible. Let him frame what hypotheſis he will for the planets, he muſt find vortexes meeting one another for the comets, that move all manner of ways. Therefore to clear this point, he muſt have his vortexes made of ſuch fluid matter, as has neither friction nor reſiſtance, and that one part be penetrable to another part: that is, ſuch a fluid as cannot be found in the world. Such is the blinded, bigotted, prejudice of ſome people, that the cleareſt demonſtrations cannot cure their madneſs. If any body aſks one of this ſtamp, how the planets move round, they preſently anſwer, in vortexes. But aſk them what cauſes the vortexes to move round, here they are at a ſtand, and have nothing to ſay, but that they move by their own nature.. But they might as well have ſaid at firſt that the planets move round by their own nature, if they can give no better account of them, and ſo ſave the labour of conſtructing theſe uſeleſs vortices.

Another author that is *vortex-mad*, is *Euler*, and he ſeems to go beyond any of the reſt, for he cannot account for the riſing of the tides without vortexes. But he has not ſhewn us by what extraordinary mechaniſm, or inviſible wheelwork his vortices are conſtituted, ſo as to be able to produce the tides, or cauſe the motion of the comets, ſince theſe vortices muſt all run counter to one another, and penetrate one another, and yet miraculouſly preſerve their motions entire.

This idle notion was firſt introduced in the time of ignorance, upon the ſuppoſition of a *fuga vacui*, or that nature abhorred a *vacuum*. But by our better acquaintance with the nature and properties of body, and the laws of motion; we now know, that the operations of nature cannot be performed in a *plenum*; and therefore a *vacuum* is abſolutely neceſſary.

If we had known of no celeſtial bodies but the planets moving all one way, the ſuppoſition of a vortex to carry them about, clumſey as it is, might have paſſed for poſſible. But one would have thought, that the comet's moving all manner of ways, would have cured this delirious notion; and have taught them the impoſſibility of ſuch a ſcheme. But theſe authors, theſe defenders of vortices, are ſo hardy, that they are not at all afraid of an abſurdity or a contradiction. Theſe things do not affect or touch them in the leaſt; but they go on unconcerned in their uſual way, tho' contrary to all the laws of nature.

Did I ſay that the theory of vortices had its original from the principle of a *fuga vacui?* I did. And the principle of a *fuga vacui* had its riſe from the phœnomena of water riſing in pumps and ſyphons, as in the *Torricellian experiment*. Here for want of trying ſuch experiments to their full extent, the principle of a *fuga vacui* was aſſumed by theſe philoſophers, as the genuine cauſe of theſe effects. So that, upon no better a footing than this, tho' the true cauſe of theſe effects (the external preſſure of the air) be now perfectly known, yet with theſe bigotted people, the ſame falſe principle ſtill exiſts.

Theſe philoſophers (if they can be called ſuch) are endeavouring, contrary to the nature of things, to find out the moſt complicate cauſes for explaining the phœnomena, inſtead of ſeeking the ſimpleſt cauſes; and they ſeem utterly to reject all ſimple cauſes, which are the greateſt beauty of nature. For if their brains were not turned round in a vortex, they could never prefer theſe complex vortical ſchemes before the ſimple doctrine of projectile and centripetal forces. Such Philoſophers!

Sir I. Newton had ſhewn in the ſchol. of Prop. 14. B. III. that the aphelions of the interior planets move a little *in conſequentia*, (by the actions of Jupiter and Saturn) in the ſeſquiplicate ratio of their diſtances from the ſun. But *Bernoulli* ſays this is falſe, for it holds not true in Saturn. And yet Sir Iſaac tells him expreſsly, it is the inferior planets that obſerve that law. So little do ſome men care what they write, if they can only put on an air of contradiction.

In finding the proportion of the axis and diameter of the earth, Prop. 19. B. III. Sir Iſaac aſſumes them to be as 100 to 101, and from thence finds the exceſs of weight, in one above the other, to be $\frac{4}{505}$ parts. Therefore he makes this proportion $\frac{4}{505} : \frac{1}{100} ::$ the force $\frac{1}{289} : \frac{1}{229}$ the true exceſs of height. Yet ſome people have objected againſt this as a wrong way. But herein they are much miſtaken; for if any force be increaſed, the effect will be increaſed in the ſame ratio; and therefore increaſing the centrifugal force here, will increaſe the difference of the heights, in the very ſame proportion; as theſe quantities are very ſmall. Upon this account, ſome people have unneceſſarily run into long calculations, and only came at laſt to the ſame concluſion.

And the ſame argument holds good againſt ſuch as have objected againſt finding the height of the tide (cor. Prop. 36. B III.) by the rule of proportion, comparing it with the centrifugal force. For all forces of whatever kind will produce *effects* proportional to the *quantities* of theſe forces; and therefore he has rightly found the height of the tide, by that proportion.

But Sir I. Newton's explanation of the tides (Prop. 24. B. III.) does not pleaſe *Euler*, tho' he accounts for every circumſtance thereof. He thinks aſcribing theſe effects to the actions of the ſun and moon, is recurring to *occult cauſes*, and therefore he had rather recur to *vortexes* for the explanation thereof; the notion of which has been confuted over and over. He denies the gravitation of bodies towards one another, becauſe he cannot diſcover the cauſe of gravity; and therefore he will not allow it to have any thing to do with the matter, as being an occult quality. But he recurs to a principle that is more than occult, his incomprehenſible vortices; which he thinks the tides are raiſed by; tho' he has not attempted to explain in what manner his vortices can do it.

He ſays, Sir I. Newton's account of the tides is not ſufficiently explained, that any certain judgment can be formed whether it is true or falſe; that he did not explain the phœnomena of the tides but only darkened them. But certainly this man does not *know* what he writes, or does not *care* what he writes. Sir Iſaac explained all the principal phœnomena thereof, and has ſhewn, that his

Theory is agreeable to obſervations both in nature and quantity. And all matters of leſs note may be eaſily deduced from his general Theory, by any intelligent perſon. But Sir Iſaac did not chooſe to erect any *pontes aſinorum.*

This author values himſelf very much for rejecting the principle of *attraction*, eſtabliſhed by *ſome Engliſhmen*, becauſe it is an occult quality. This profound Philoſopher will either know every thing or nothing; he will make no uſe of the forces of gravity, becauſe he knows not the cauſe of gravity. For the very ſame reaſon he would want to know the cauſe of that cauſe; and ſo he muſt know every thing in the whole chain of cauſes and effects, or he cannot be ſatisfied. This odd temper in ſome men ariſes from the pride of the human mind, which attempts to ſoar above its ſphere. They diſdain to know the little matter that is within their power to know; whilſt they are continually aſpiring at things that are without the reach of their knowledge, things for which they have no *faculties* ſuited to underſtand, and no *data* to determine. This is not to philoſophiſe, but to trifle.

As this Gentleman takes the liberty to ſneer at the *Engliſh*, he may pleaſe to ſtay and take this obſervation along with him, that if it had not been for an *Engliſhman*, he (and ſuch like) had known nothing. To him they are beholden for all theſe great and wonderful diſcoveries, which all the world acknowledge and praiſe him for, except theſe ſons of detraction, envy, and ingratitude. Diſcoveries that may be looked upon as the works of a Genius rather divine than human.

The *Engliſh* have found by experience, that by the power of ſome cauſe or other, all bodies are drawn or impelled towards one another, according to certain laws. And theſe laws are found out by obſervations, and a juſt reaſoning from the laws of motion. This cauſe, be what it will, they call *gravity*, without pretending to determine of what kind it is. If this author knew the phyſical cauſe of gravity never ſo well, he would be no wiſer; for the effects of it would be juſt the ſame; and we can meaſure the effects of gravity, and from thence find the quantities, and proportions of the generating forces, without knowing what theſe forces conſiſt in. So that the knowledge of ſuch a cauſe would be only a uſeleſs ſpeculation. But this author is ſo fearful of occult cauſes, that he dare not make uſe of their manifeſt effects; and condemns the Engliſh at once for believing their own eyes, that there are ſuch effects flowing from this unknown cauſe, which they have agreed to call *gravity*. For it appears to them, that the effects thereof are all that they have any thing to do with. Sir I. Newton tells him more than once, *that he does not take upon him to define the kind or manner of action, or the cauſes or phyſical reaſon thereof*; but this author cannot take it in.

Some perſons, when they can find nothing elſe to ſay, have cavilled about the term *gravity*, and ſo their objections are dwindled to a diſpute about words, and not things. I ſuppoſe every thing in nature, that we have occaſion to make the leaſt uſe of in our diſcourſe and reaſoning, be it known or unknown, ought to have a name given it, to diſtinguiſh it from other things, and to convey our meaning to others. And thus the *cauſe* of the acceleration of bodies towards one another is expreſſed by the word *gravity*, and the action itſelf is called *gravitation*. The word *attraction* is uſed in the very ſame ſenſe; that is, not in a *phyſical* but *mathematical* ſenſe, which regards only the *quantity* of the cauſe. That theſe terms are the moſt proper that could be choſen, will appear from hence; the word *attraction* is taken from its moſt manifeſt quality; for a body dy moving towards another with an accelerated motion, has all the appearance poſſible of being drawn towards that other by ſome inherent virtue in the other. The word *gravity* has evermore expreſſed the tendency of a heavy body to the earth; and therefore by parity of reaſon, will as properly expreſs the tendency of the moon towards the earth; or of the earth towards the ſun, or of any body towards another body.

Suppoſe this philoſopher, or any other of that ſort, was aſked to calculate the times, ſpaces, or velocities of falling bodies; which is the moſt ſimple caſe that can be propoſed about gravity. Would not any body juſtly laugh at him, if he ſtood to demur about it, and refuſe to calculate, till he knew whether their falling was cauſed by attraction, impulſion, or the rotation of a vortex? And would not he equally deſerve to be laughed at, that ſhould heſitate to calculate the motions of the moon, or of the earth and planets; or of the tides; upon the ſame account; when they are all acted upon by the ſame unknown cauſe of gravity?

But to return to the tides. This Gentleman (*Euler*) tells us, that Newton's method is erroneous, by which he found the ſea to riſe to the height of near two feet, by the ſun's force only. And ſays, that Newton found out this enormous effect, by comparing the ſun's force with the centrifugal force of the earth. This has been anſwered before; and certainly this Gentleman knows little about the nature of forces, if he does not allow that two equal forces, of however different kinds, will always have equal effects; and proportional forces, proportional effects, eſpecially in their naſcent ſtate. For it is not the *kind*, but the *quantity* of force that is to be regarded. Therefore Newton rightly found the ſolar tide near two feet, and the lunar tide $8\frac{1}{2}$ feet, agreeable to experience. But to ſhew you what ſort of a theory this Gentleman works by, he finds the ſolar tide only half a foot, and the lunar tide $2\frac{1}{4}$ feet, in all not three feet; which all obſervations confute, and with it, his erroneous method of computation. I have met with no body yet, but what makes it at leaſt three times as much.

He alſo tells us, that Newton found out the forces of the ſun and moon by help of the tides, but he has not done it accurately. And yet Newton took in every circumſtance that could any way affect it; as may be ſeen in Prop. 37. B. III.

Having had occaſion to compare different kinds of forces with one another; I will venture to lay down this as a general Rule, that all forces whatever, whether attractive or impulſive, centripetal or centrifugal, or of what kind ſoever, if they be equal, they will produce equal effects. And therefore how idle muſt it be for theſe men to wrangle about the *kind*, when the *quantity* only is concerned in the effect, and can only be of any real uſe to us in our calculations. The enquiry after the *kind* and *modus* of action, is a phyſical, or rather a metaphyſical ſpeculation, the knowledge of which they can never come at.

It has likewiſe been objected by ſome perſons, that the two examples of Newton, for finding the tides, are ill choſen. But however, he had no more to chooſe on, and by their near agreement, it ſhews they were well choſen. *Euler* tells you, that at *Havre de Grace*, the greateſt and leaſt tides are as 17 to 11; and therefore the ſun's force to the moon's, will be as 17 — 11 to 17 + 11, or as 6 to 28; or as he makes it, as 7.13 to 28, which is about as 1 to 4, a proportion not very different from Newton's. *Dan. Bernoulli* ſays, that at St. *Malo*'s, the greateſt height to the leaſt, is as 50 to 15, which makes the ſun's force to the moon's, as 35 to 65, or as 7 to 13, not ſo much as 1 to 2; a concluſion utterly inconſiſtent with all other obſervations;

ſervations; which argues, that the obſervation has not been made with ſufficient accuracy. However this is certain, that if any place can be improper for ſuch an experiment, this place is; by reaſon of the very extraordinary tides. For here the tide being hurried up a long channel, growing continually ſtraiter, it is forced up to an unuſual height.

However, I cannot think that it ſignifies a great deal, where, or in what places, theſe experiments are made, provided the ſea be deep, and have free acceſs and receſs to and from the place of obſervation. For tho' the tides be higher in one place than another, the ſun and moon conſpire alike to that; for if the water be accelerated in any degree by the moon's force, it will likewiſe be accelerated proportionably by the ſun's force; ſo that the reſult will be nearly the ſame.

I cannot but be ſurpriſed, that in ſo material a point as this, no body has been ſent purpoſely, to proper places to make obſervations of the tides. Since by this method only, the forces of the ſun and moon can be determined to any tolerable degree of exactneſs. No celeſtial obſervations can aſſiſt us in this matter. Aſtronomy affords us no help. And there ſeems to be no way for us, to gain this great point, but this method by the tides. The forces of the ſun and moon are ſo very ſmall, in reſpect to the force of gravity, that no common hydroſtatical experiment can ſhew us the leaſt effects thereof. It is only in the tides that their effects become ſenſible.

The forces of the ſun and moon reach to the very center of the earth, and act upon every point of the radius or column of water under them; and diminiſh the gravity of every particle thereof; and all theſe forces conſpire together to raiſe the tide; and therefore the radius of the earth becomes the proper ſcale for all theſe forces to act on. And we cannot ſee the total effect of all theſe forces, unleſs we have a depth equal to the whole radius of the earth, or an extent of ſea of 90 degrees; and this effect or the tides, when at the greateſt, will then only amount to a few feet.

If I was to give directions for making obſervations of the tides for this purpoſe, I would adviſe to chooſe ſome place near the equinoctial, as on the coaſt of *Africa* or *America*. And the place would be moſt advantageous, where the ſun and moon are in the zenith at the firſt obſervation; and in the horizon, at the ſecond, for the ſpring tides. And one in the zenith and the other in the horizon, for two obſervations at the neap tides. And ſuch places ſhould be choſen where the ſea is of large extent, and deep; ſo as to communicate freely with the place, back and forward. And ſuch places will be beſt in a calm country, and performed in calm weather; and all circumſtances of weather ſhould be alike (as near as can be) at any two correſpondent times of obſervation, the ſyziges and quadratures. Theſe obſervations ſhould be made at the ſyziges and quadratures, both day and night. And alſo when the tides are higheſt and loweſt, which is three days after. But for ſome purpoſes, trial ſhould be made every day. And the buſineſs of trying at the ſyziges and quadratures, ſhould be continued for a twelvemonth. And more places than one ſhould be tried. Perhaps ſome iſlands in the middle of the ſea, as *St. Helena*, may be proper; for the weather is likely to be more uniform, than on the continent. Having thus gained a ſufficient number of obſervations, the beſt may be ſelected, and a mean ratio found, by which this matter will be finally determined.

There are ſome people that object againſt this method of finding the ſun and moon's forces, by the tides; and reckon it very precarious, and ſubject to many obſtacles and intervening cauſes, by which the tides are perpetually influenced and diſturbed; as if every thing had not its difficulties; the only diſturbing cauſe is the wind. Yet they can tell us of no other method, but what is more precarious and more impracticable, and leſs exact. And ſo much for the tides.

The 39th prop. is about finding the preceſſion of the equinoxes; againſt this, being a prob. of great difficulty, objections have been raiſed by ſeveral people, alledging that it is not truly demonſtrated. Mr. *Simpſon* is among this clan; and he abſurdly makes the preceſſion by the ſun's force alone to be 21″ 6‴, which is double to Sir I. Newton's; the conſequence of which is, that the motion by the moon's force will be only 29″. So that by this, the moon's force will be to the ſun's, only as $1\frac{2}{5}$ to 1; yet he ſays in another place, that the moon's force to the ſun's cannot be leſs than $2\frac{1}{2}$ to 1. So inconſiſtent and erroneous are his operations. But this has been taken notice of before in the Comment. But the moon's force muſt be greater than $2\frac{1}{2}$; for in all obſervations of the tides, that have been regularly made, the moon's force is 4 or more. And it is hardly poſſible in any obſervation of the tides, made with any tolerable degree of care, to miſs near a half. They that would ſee the preceſſion of the equinoxes truly calculated, will find it in Prob. 26, Sect. III. of my Fluxions.

Other objections of leſs moment I paſs over, as their malignity and falſhood will appear to every reader. Such as the abſurd opinion that motion cannot be loſt in the world; (concerning which, ſee my Mechanics, 4to. prop. 10. and cors.) Alſo the falſe opinions of theſe that deny that the compoſition and reſolution of forces, are analagous to the compoſition and reſolution of motions, which are their adequate effects. Their endleſs cavils againſt his method of demonſtrating any propoſition, which is the ſynthetical; preferring their own analitical methods as being ſhorter, which is no wonder. Their introducing into phyſical calculations, an obſcure, precarious law or force, called *vis viva*; founded upon no certain principles. Concerning which ſee my Mechan. 4to. ſchol. to prop. 11, where the nature of it is unravelled. Their promiſcuouſly uſing the words force and motion, for one another, which are as different as cauſe and effect; which induces no ſmall obſcurity into their writings. Their aſcribing the invention of *Fluxions* to *Leibnitz*, contrary to the cleareſt teſtimonies; and ſome of them tell us, that, *not he that firſt found, but he that firſt publiſhed it, deſerves the praiſe.* As if the publiſher could have publiſhed it, at all, if the inventor had not firſt found it out. Here the inventor is robbed of his due praiſe, to give it to the thief that ſtole it. And in general, their aggravating every trifling ſlip, as a capital crime; and inſtead of praiſing him for what he has done (which is more than all the world ever did before) they diſpraiſe him for what he *has not* done, or had not time to do; and lampoon him becauſe his time or his knowledge was *not infinite.*

But if any body ſhould aſk, what any of theſe *Bravo's* have done ſince? The anſwer is, nothing at all, or leſs than nothing; they have been turning ſcience backwards. For they have been doing nothing but undermining his principles, (tho' built upon the ſureſt foundations,) to introduce their own chimerical hypotheſes, that have nothing to ſupport them but impudence, ignorance and preſumption.

At the end of the Principia, Sir I. Newton has given us his thoughts of the deity. Here he ſhews that God is an *eternal, infinite*, and *powerful being.*

being. That all the frame of nature is owing to him, which he made, and governs. And from the ſimilitude of all the parts of the world, he ſhews that God is *one*. That he is a being acting with *council* and *deſign*, and with the greateſt *wiſdom*. That we have ideas only of his *attributes*, but know nothing of his *ſubſtance*; nor after what manner he acts or does any thing. Indeed we know nothing of the ſubſtance of any thing, much leſs of God. And as we cannot conceive of ſpace and time, but as neceſſarily exiſting; much more muſt we allow that God exiſts neceſſarily; and conſequently that he exiſts *always* and *every where*; and that not *partially* but *totally*. In ſhort, he has laid down the beſt metaphyſical notions of God, that can be met with any where.

But M. *Leibnitz* will not allow that God has in himſelf (much leſs other animals), any principle or power of acting, but as he is firſt acted on by ſome *motive*, which he thinks is to determine his actions. And in conſequence of that, if two equal ways of acting were laid before God, he could chooſe neither; but be like a balance, acted on by two equal weights, that would turn no way. But at this rate God is not a *free agent*, but a mere *patient*. He reckons it an *imperfection* in God to be able to chooſe *one* out of two perfectly equal things; becauſe he ſays there wants a *ſufficient reaſon*; and therefore he can chooſe neither. But it is certainly a *greater imperfection* to chooſe *neither*, than to chooſe *one*. It is a principle with him that God muſt have a ſufficient reaſon; which is true, but it is God's mere will that is the ſufficient reaſon. And how comes he to know what is a ſufficient reaſon to a perfectly free agent, that has it in his power to do any thing; as if man, who is no more than a *worm* in the creation, can preſume to know what is fitteſt to be done.

Sir I. Newton has aſſerted, that all the great bodies in the world move in free ſpace, which is unbounded and infinite. That all bodies have pores or empty ſpaces within them. But this author will not allow any *vacuum*, but will have the world be a perfect *plenum*. And he tells us alſo, that the world will continue for ever without any alteration (or mending); but he has taken particular care to prevent that, by introducing his *plenum*. The motions of the heavenly bodies muſt needs be retarded and ſoon ſtopt, in moving thro' denſe matter, tho' never ſo fluid. So that one of his ſuppoſitions is inconſiſtent with another; and no body could more effectually deſtroy his own hypotheſis, than he has done himſelf. He had had infinitely better chance for this ſuppoſition, if he had made the planets, &c. to move in a vacuum.

If God had deſigned every thing to ſtagnate, and be fixt in the world; then a plenum ſeems to be the beſt conſtitution for that end. But as all the operations of nature are to be performed by motion, which indeed is the beauty of nature; then placing them in vacuo, muſt needs be the beſt conſtitution, as the motions will then be free and unreſiſted, and the moſt durable poſſible. And therefore it is as neceſſary to have a *vacuum*, in which theſe motions are to be performed, as it is neceſſary to have *body* to perform theſe motions.

The ſame objections he has made againſt empty ſpace, he makes againſt time; for with him ſpace is no more than the order of co-exiſting beings; and time the order of ſucceſſive ones; but at this rate the words *nearer* or *further*, *ſooner* or *later*, ſignify nothing; yet both ſpace and time are meaſured by quantity, and therefore are themſelves quantities and conſequently real beings, which confutes his notions. And indeed common ſenſe is enough to determine this; for their exiſtence is ſelf-evident, even to the moſt ignorant; and cannot be made more evident by all the arguments in the world. Arguments in ſuch caſes, ſerve for nothing but perplexing things that are plain of themſelves, and are often brought for that purpoſe. *Actions* and a *vacuum* are the firſt principles of the moſt ancient Philoſophy; which is the ſame thing as to ſay, that God has made *bodies*, and *room* for them to act and to move in. For the whole world appears to our ſenſes, to be nothing but *matter* and *motion*.

This learned Philoſopher's argument againſt empty ſpace is this; *every perfection which God could give to things, without derogating from their other perfections, he has given them. Suppoſe then an empty ſpace, God could have placed matter in it, (which is more excellent than empty ſpace); therefore he has done it, and conſequently there is no vacuum.* Here he ſuppoſes that there is no excellency or perfection at all in motion; and ſo this world-maker will not agree to leave any room or free ſpace for bodies to move in. And in this he is as dogmatical, as if he had been originally one of God Almighty's privy council. But had he gone one ſtep further, he would have ſeen the abſurdity of it. For God did not make the world to ſtand ſtill; but to move after various laws and rules. And conſequently when this acute Philoſopher fills the planetary regions with matter, he *derogates from the perfections* of the other bodies, by deſtroying their motion, with his new matter. And in this caſe, and for this very purpoſe, empty ſpace is more excellent than matter.

But the *ignis fatuus* that leads him aſtray, is the argument of *a ſufficient reaſon*; when at the ſame time he is no judge at all what is ſufficient. Yet this principle he brandiſhes about with no ſmall oſtentation and aſſurance. But it is of no more uſe in his hands, than the ſword of *Achilles*, in the hands of an infant.

CONCERNING THE

OPTICS.

NO ſooner did Sir *I. Newton* publiſh his theory of Light and Colours, than he was attacked on all ſides. A doctrine ſo ſtrange, and contrary to all the received opinions about the nature of light, could not be admitted. This brought him into many frivolous diſputes, with injudicious people that had imbibed other principles. Yet he ſhewed the truth of his theory, by undeniable experiments, if his oppoſers had but had underſtanding enough to conſider them. He had read lectures before, in the Univerſity of *Cambridge*, concerning light and colours, which he was thinking of publiſhing; but the diſputes he was involved in, from what he had publiſhed in the Tranſactions, made him lay aſide the thoughts of publiſhing any more. And ſo theſe lectures were laid by in the Univerſity, and were not printed till after his death. And in this I think he was much in the right; for who would ever ſpend any time or labour, to make men wiſe againſt their wills; and eſpecially a ſet of men, that loved cavilling and diſputing better than truth and knowledge.

By reaſon of theſe ſenſeleſs diſputes, he would not agree to print his book of Optics for many years; and at laſt it was only printed in Engliſh, with a deſign it ſhould not go abroad, to create any freſh diſputes; but was afterwards tranſlated into Latin.

This book, tho' full of the moſt curious and profound diſcoveries in *Optics*, could not paſs without the moſt ſevere ſcrutiny; and I believe, hardly a propoſition in the book but what was diſputed by ſome raſh Critic or other. But as the proofs were moſtly experimental; as ſoon as people had a mind to leave their prejudices, and try theſe experiments fairly and impartially; they found them all true and agreeable to his theory, which at length began to gain ground; and all diſputes are now ſubſided.

As there are ſome things laid down in this book, without their demonſtrations; ſuch of them as are difficult, are calculated in my book of Optics.

Towards the end of this book Sir Iſaac tells us, the true method of philoſophiſing, his thoughts of the original and conſtitution of the world, and of the Deity. He tells us the world could not ariſe out of a *Chaos*, by the mere laws of nature, but muſt be the work of a powerful, everliving agent, acting with wiſdom and deſign. He tells us, that it appears from the phœnomena, that *there is a being incorporeal, living, intelligent, omnipreſent; who in infinite ſpace, as it were in his ſenſory, ſees the things themſelves intimately, by their immediate preſence to him ——whilſt the images only of theſe things are carried thro' our organs of ſenſe into our little ſenſoriums, and are there ſeen by that which in us perceives and thinks.*

But Mr. *Leibnitz* catching at the word *ſenſorium*, and perverting Sir Iſaac's meaning, tells you, that Sir Iſaac makes ſpace to be the ſenſorium of God, as an organ, by which he perceives things, and thereby makes God the ſoul of the world, contrary to Sir Iſaac's plain declaration; whereas he ſpeaks only by way of ſimile (*as it were, in his ſenſory*). For ſuch is the imperfection of our notions concerning God, that when we would deſcribe any of his properties or attributes, we can only do it by ſimilitude, comparing it with ſomething ſimilar to it in ourſelves; which ſurely cannot be taken in a ſtrict ſenſe. Nothing is more common than to talk of ideas in the mind of God, when we ſpeak of his knowledge or omniſcience. Yet it is certain God has no need of ideas or images (as we have), when he is preſent to the things themſelves. And I think it was hardly poſſible for Sir Iſaac to find out a fitter ſimile, than to compare *infinite ſpace* with the *place* where *God* perceives all things; juſt as the brain is the place where *men* perceive the images (only) of things. For ſuppoſe the brain to be the *organ* of perception, it is nevertheleſs the *place where* theſe images are perceived. This miſconſtruing the words of Sir I. Newton, ſhews the captiouſneſs of the man; eſpecially when he is told that God is *a uniform being, void of organs, members or parts, that all things in the world are his Creatures, ſubordinate to him, and ſubſervient to his will. That he is no more the ſoul of them, than the ſoul of a man is the ſoul of the ſpecies of things, in the place of his ſenſation.* All this, one would think, ſhould ſtop his mouth.

This Gentleman calls God a *ſupramundane intelligence*, which is only an unintelligible term to quack with. For certainly he exiſts *in* the world, but after what manner is a ſecret to us. If God have any exiſtence, he exiſts *in* ſpace; to ſay that he exiſts *out of* ſpace, is to ſay that he does not exiſt at all, becauſe ſpace is every where. So does he exiſt *in* time; for to exiſt *out of* time, or in no time, is not to exiſt. He exiſts then in all ſpace and in all time, that is *always* and *every where.*

Sir Iſaac tells us blind fate could never make all the planets move one and the ſame way, in concentric orbs; ſome inconſiderable irregularities excepted, which may have riſen from the mutual actions of comets and pla-

nets upon one another, and which will be apt to increaſe, till this ſyſtem wants a reformation.

Now theſe actions of the planets and comets on each other, and the conſequent irregularities produced thereby, are matters of fact, and evident from obſervations. And it is as evident that they will increaſe, from the ſame cauſes. Yet Mr. *Leibnitz* is ſo ignorant of theſe effects, that he takes upon him not only to cenſure Sir I. Newton for making theſe obſervations, but God Almighty for not making his work otherwiſe; and compares him to an *imperfect workman*, that has his *watch* to wind up now and then. He thinks it an imperfection in God, if he does not make the world ſo as to laſt in the ſame ſtate for ever. At this rate he may as well think it an imperfection in his Maker, if man does not laſt for ever, and live for eternity.

It is a bold aſſertion to ſay that God cannot make any thing to what degree of perfection he pleaſes, or to laſt as long as he pleaſes, or to reform, regulate or alter it at his pleaſure. Nothing will pleaſe this Gentleman but to make God to act *ad extremum*, as all neceſſary agents do, if they can be called agents. At this rate, not only the world muſt be made ſo perfect, as to laſt for eternity, and want no regulating, which is the thing he affirms; but even the bodies of men and other animals, for the ſame reaſon as was obſerved before, muſt laſt for ever, and never come to decay, or need any repairs; which is arguing againſt the cleareſt evidence of his ſenſes; and is a demonſtration of the abſurdity of his hypotheſis. So that in ſome caſes it is true (what ſome men have objected againſt ſcience in general), that the greateſt philoſophers are the greateſt aſſes.

Go, teach eternal wiſdom how to rule,
Then drop into thyſelf, and be a fool.

Pope.

His Doctrine of *Indiſcernibles* is altogether as wild a notion as the other of forcing God to do his beſt. His principle is this, that there are not in nature, two *real* abſolute beings, *indiſcernible* from each other, (that is, two ſuch things as are perfectly alike). For if there were, he ſays, God and nature would act without a reaſon; it is a thing contrary to the divine wiſdom, and therefore they do not exiſt. And he inſtances in two leaves or two drops of water. And he ſeems to build his opinion upon obſervation, which is, that if you ſeek never ſo long, you cannot find two leaves alike; and upon this he raſhly concludes, that it is impoſſible; becauſe God would not be wiſe in doing of it. However, if he can give no wiſer a reaſon than this, I doubt his principle will fall to the ground. For in the nature of things, what antipathy can God have againſt two equal or like things, any more than againſt two unequal or unlike things. It is impious thus to tie the hands of God, and vain in him to pretend to know what God's wiſdom will chooſe or will not chooſe. His attempt to know this is as ridiculous as it would be, in attempting to move the whole world with the ſtrength of a finger.

I believe the reader will hardly be ſatisfied, whilſt I am endeavouring to confute his notion if I do not advance a better ſolution. This I ſhall attempt as follows, by a very ſimple and plain caſe. Suppoſe the length of the leaſt ſort of leaves to be a, of the greateſt b; their difference $b-a$ or d. Now I hope it will be allowed me, that the quantity or length d is infinitely diviſible; and conſequently there is an infinite number of different lengths contained in the length d; and hence there may be an infinite number of leaves, and all of different lengths, in a gradual increaſe from a to b. This being ſettled, let us conſider the number of leaves. The number of them on any one tree is but finite, and the number of all the trees in the world is but finite; for the ſands of the ſea are numerable. Therefore all the leaves in the world make but a finite number. Suppoſe then any one length be named for a leaf; it will be the odds of infinity to 1, that in a geometrical exactneſs, there is not a leaf of that length in the world; becauſe the number of *leaves* being ſhort of the number of poſſible *caſes*; there will be an infinite number of caſes that do not happen; and therefore it is infinity to 1 againſt the happening of any one in particular. It follows likewiſe that, name the length of any one leaf, and it is the odds of infinity to 1, that another cannot be found *preciſely* of the ſame length.

Now this calculation regards only the length, but there are other circumſtances as variable, ſuch as breadth, weight, colour, form, curvature, different arrangement of the veins, &c. and all diverſified in quantities and ways infinitely various; therefore the odds will, by this means, be infinitely multiplied; ſo that it is next to impoſſible for ſuch a thing to happen naturally. And here, we need not to have ſuppoſed infinite degrees of length; for if we had but aſſumed a very great number of lengths, each differing by the leaſt conceivable quantity; yet by the infinite number of circumſtances that attend it, the reſult would ſtill ſhew that it is almoſt as infinity to 1, againſt ſuch an event happening. And by the ſame reaſoning one cannot find two ſtones alike, or two drops of water.

As to drops of water, there is not as great a variety as in leaves. Yet if you ſuppoſe but one heterogeneous particle; that ſingle particle is capable of an infinite number of ſituations; and there may be 2, 3, &c. or any number of ſuch particles;

from all which, the ſame concluſion will happen as before. And here we may obſerve, that the more compounded any body is, the greater is the improbability of this event happening.

Hence appears the true reaſon why ſuch events do not happen, and that is, the infinite number of *poſſible* caſes, and the finite number of *real* caſes; and upon the infinitely *ſmall* chance there is for its happening; but it is not at all from the impoſſibility of the thing; which may be made plain by this inſtance. Suppoſe there be 100 dice, and any one ſhould undertake to throw all the aces at once. The number of chances is ſo great againſt him, that in all probability, he would never do it while the world ſtands. Yet the thing is not impoſſible in itſelf; for the dice may be ſo directed by art, as to be thrown every time.

So now the whole myſtery is diſcloſed that ſeduced this learned Philoſopher to abridge the *power* of God, for fear of contracting his *wiſdom*. You may ſee that all the ſizes or forms of leaves are alike indifferent to God, and nature has the ſame chance for any length or form. But what is all this to do with the *power* of God? He can certainly make millions of things all equal and alike, if he has need of them in the creation, as eaſily as he can make them unequal or unlike. But the excuſe this Gentleman has for this chimerical opinion, is this, he ſays *two things* perfectly alike will not be two things, but only *one thing*. But this is an aſſertion equally ridiculous, for two things will be as diſtinct, if they differ but in one property, as if they differed in ten thouſand. Nothing can identify two things (*i. e* reduce them to one), but exiſting in the very ſame place, at the very ſame time, and having all other properties the ſame.

Men are ſuch imperfect artiſts, they cannot pretend to make things equal or alike, to a geometrical exactneſs; but they can make them ſo near as they cannot diſcover any difference. But God can work perfectly, which men can only imitate.

God is a being ſelf-exiſtent, omniſcient, omnipotent, and omnipreſent. This being can do all things that are poſſible to be done, or that imply no contradiction. Theſe are his natural attributes, as has been mentioned before; and which Sir I. Newton has given a ſublime deſcription of, both here and in the Principia, which this *babbler* has thought fit to criticiſe, but with an ill grace. As to his moral attributes, wherewith ſome people have inveſted and complimented God Almighty; as his *Goodneſs*, *Mercy*, *Juſtice*, &c. theſe are not real properties, but are only conſequent from, and included in, his wiſdom. As for example, God cannot be ſaid to be infinitely good, as he may be ſaid to be infinitely powerful, or infinitely wiſe; but he is always ſo far good, as is conſiſtent with his wiſdom; and the like for his mercy, &c. As to our knowledge of God, we muſt content ourſelves with the knowledge of his natural attributes; for we know nothing further relating to him. It is a profound ſecret to us how he acts, how he perceives, or how he exiſts. Here he hides himſelf from us and will not be known. Nor ſhould we even know that ſuch a being exiſts at all, but from our own exiſtence, and the regular phœnomena that appear in the world, which bear witneſs to it. So that we are ſure there muſt be a firſt cauſe, from whom and by whom all theſe things have their original; as is ſhewn at large by our great author Sir *I. Newton*.

I believe the improvements that have been made to the *Science of Optics*, ſince he left it, are few or none; his queries are ſtill to ſeek, no body has inveſtigated any of them. One would think that theſe cavillers againſt gravity, as an *occult quality*, would have taken care to have inveſtigated its cauſe, hinted at in theſe queries; but it remains as occult as ever, for them. They are more ready to diſpute than to act. To compleat theſe things, we ſhould have another *Newton*; but as that is not to be expected, we need not think that any improvements will go faſt forward.

As I have conſidered the principal objections that have been made againſt the *Principia* and the *Optics*, moſtly by foreigners. It cannot be amiſs if I take notice of ſome that have been made againſt his *Chronology*, by ſome in our own country; to ſhew how they *bite the file*.

An Account of ſome of the numerous Inconſiſtencies, contained in the Objections made by the Rev. Dr. Rutherforth, *Regius Profeſſor of Divinity in the Univerſity of* Cambridge, *againſt Sir* I. Newton's *Account of the Argonautic Expedition.*

Eſt genus hominum qui eſſe primos ſe omnium rerum volunt, nec ſunt :

Terent. *Eun.*

THE Profeſſor ſets off by telling us, that the *whole of his (Sir Iſaac's) argument, for ſettling the time of the Argonautic expedition, depends upon three principles.* 1. *That Chiron and Muſeus made a ſphere.* 2. *That they drew the Colures of the equinoxes thro' the* 15*th degrees of Aries and Libra, and that of the Solſtices thro' the* 15*th degrees of Cancer and Capricorn.* 3. *That they knew how to draw the Colures exactly.*

Here he begins by aſſerting a manifeſt falſhood; for the *whole* of his argument does not conſiſt in theſe three Articles. For Sir Iſaac ſtrengthens his argument by ſo many events, taken out of hiſtory, which all conſpire to prove the ſame thing, that no body (except they be ſtrangely bigotted, like this Gentleman) can refuſe his aſſent. But let us examine his three grand principles. 1. *That Chiron and Muſeus made a ſphere.* Now this is a point of hiſtory, and therefore ought to be taken as matter of fact, or elſe there is an end of all reaſoning from hiſtorical relations. Yet our Doctor ſhews a ſtrong inclination to ſuſpect its truth, if it could be done; and ſeems to waver between two opinions, whether it was, or was not ſo. However, not being able to get over that, he tells us, that theſe figures drawn on the ſphere, muſt relate to religion; *for delineating conſtellations for the uſe of ſailors, could be of no ſervice for the practice of juſtice.* Here one would expect that he ſhould have ſhewn us how delineating conſtellations *not* for the uſe of ſailors, could be of any ſervice to the practice of juſtice; or what relation there is between *juſtice* and the *conſtellations.* But unluckily he has left this ſhort. Now every body knows the uſe of the conſtellations in ſailing: they had no other way at that time to guide their ſhips thro' the ocean, but obſervations of the ſtars. This uſe then is obvious; but the other uſe, which the Profeſſor dreams on, is to ſeek. As to *Hippo* the daughter, who is allowed to have ſtudied nature in the ſame manner as the father; the Dr. ſays, ſhe ſhewed the will of the Gods, by her oracular anſwers, or *from the riſing of the ſtars.* The uſe ſhe made of her knowledge, was to teach the will of the Gods *from the motion of the ſtars.* This is enough ſurely to ſhew that they both ſtudied *Aſtronomy.* But then this knowledge was applied, it ſeems, to the purpoſes of *Augury* or telling fortunes. But can any body (but this Rev. Doctor) be ſo dull, as to think that two of the greateſt Aſtronomers of the age ſhould apply ALL this knowledge, to ſuch a trifling inſignificant purpoſe; and not rather apply *ſome* of it leaſt, to the nobler purpoſe of *Navigation,* for which it is naturally adapted, and very much wanted for that purpoſe. For without the knowledge of the ſtars, they could make no long voyages;

ages; nor had they any other method at that time to guide a ſhip, but the help of the ſtars. Conſequently it was natural for the navigators to apply to the aſtronomers for that purpoſe. But the Profeſſor ſeems ſtill not to be content in allowing theſe figures to be the conſtellations; for, ſays he, *perhaps this may not mean to form conſtellations, but to divide the heaven into regions.* As if dividing the heaven into regions or ſegments, could teach the *riſing and motion of the ſtars. Muſeus,* he ſays, *was a poet, and writ a poem concerning the ſphere, but there was no ſort of reaſon to conclude that he was a practical aſtronomer.* I dare ſay, no body would ever draw ſuch a concluſion, but this ſagacious Doctor. For can any body in their ſenſes ſuppoſe, that any man will take pains to write a poem concerning a ſubject, that he knows nothing at all about. How muſt he deſcribe the circles of the ſphere, or ſhew the nature and uſe thereof, if he did not underſtand the ſphere? But all thinking men muſt rather believe that he knew what he was doing, that he underſtood the ſphere, and of conſequence muſt be an aſtronomer. I ſay, the conſequence is plainly this, that there was a ſphere in the time of *Muſeus,* or he could not write about it, and that he underſtood the ſphere he was deſcribing; and if he alſo made a ſphere, as hiſtory informs us he did, and was the firſt that made one; then this point is fully determined. But if it was only a poem, yet certainly if a man writes upon a ſubject (eſpecially a mathematical one) he muſt be allowed to underſtand that ſubject, whether he writes in verſe or proſe. Yet the Rev. Doctor will not allow *Muſeus* to be an aſtronomer, becauſe he was a poet; which is juſt as good a reaſon as this, Dr. Rutherforth is a *Parſon,* and therefore is no *Aſtronomer.*

He ſays further, *the accounts we meet with in the* *poets* (concerning the Argonautic Expedition) *is only its fabulous hiſtory. And the names of the conſtellations Sir Iſaac has taken from this fabulous hiſtory.* The poets then cannot be truſted, even to give the names of things truly. But this writer is certainly at liberty to call any thing fabulous he pleaſes. He may reckon the whole fabulous, if he likes; and that the expedition itſelf is nothing but a fable. The poets give us a mixture of truth and falſhood, or fables intermixed with truth. This Sir Iſaac knew, and by his *philoſophical ſieve*, could tell how to ſift out the true, and leave the fabulous part behind. Whilſt ſuch writers as this Profeſſor are forced to take altogether by the lump; and not knowing how to ſeparate them, call the whole fabulous. However, what Sir *I. Newton* has taken, is moſtly from the Hiſtorians, who muſt be ſuppoſed to relate matters of fact, and not fables; and theſe he corroborates from the teſtimony of the poets. And as far as I can find, this Doctor is of opinion, that the Conſtellations were formed without having any names given to them; or elſe what ſhould hinder the names from being tranſmitted down to poſterity, as well as the Conſtellations themſelves. Nay, they could not be deſcribed without their names.

The next objection this Rev. Doctor makes is, placing the conſtellation *Argos* in ſuch a poſition, that the principal ſtar *Canopus* could not be ſeen; and he thinks, *it ſhould rather have been placed in the zodiac.* But any body may conſider, that the conſtellations in the zodiac are of far greater uſe than any of the reſt, as they ſerve to ſhew the motions of the ſun, moon, and all the planets; and therefore theſe muſt be firſt of all placed; and this being ſo, he could not place *Argos* in the zodiac without diſplacing ſome of the reſt, that had been fixt before; conſequently it muſt be placed where he could find a vacancy. In the poſition ſhe was placed, her maſts, ſails, and rigging might be ſeen. Beſides, it is evidently more natural to place the ſhip in the horizon, with part of her inviſible, as if under water, than have her ſuſpended in the air; as ſhe would have been, had ſhe been ſet in the zodiac, where ſun, moon, and planets move. But let us come to his ſecond principle.

2. *That Chiron drew the colure of the equinoxes thro' the 15th degree of Aries and Libra*, &c. Now this is a point of hiſtory, which is all the *data* allowed, and therefore muſt be granted. If this Profeſſor diſputes it, then he is diſputing with *Eudoxus*, and not with Sir *I. Newton*; for *Eudoxus* was deſcribing the ſphere of the antients. But this Doctor tells us, *that he was deſcribing the ſphere in his own time*; the conſequence of which is, that *Eudoxus* (and even *Thales*) muſt have lived 940 years before Chriſt, which is a manifeſt Falſhood. He tells us, that in deſcribing the colure of the ſolſtices, Sir Iſaac *leaves off, and omits a very material part, which Hypparchus adds.* This ſagacious Doctor, it ſeems, did not ſee the reaſon of this; Sir Iſaac had *data* enough without going any further, to determine what he wanted. And if the remaining deſcription was as it ſtood in *Eudoxus*'s time (as this writer ſays); then what had Sir Iſaac to do with that? for the poſition of the colures muſt be quite altered in near 600 years. But the Doctor ſeems not to underſtand, that many of the conſtellations had been miſplaced, and drawn otherwiſe in ſucceeding ages, and could not well be rectified; and ſuch as were rightly placed, or could be rectified, Sir Iſaac made uſe of. Beſides, if this Objector knew any thing in Aſtronomy, he would eaſily underſtand, that ſuch ſtars as are placed near the pole, or far from the ecliptic, are ſubject to great errors; for a ſmall one there, becomes a

very great one at the ecliptic, and therefore were judiciouſly omitted by Sir Iſaac; and for this reaſon, he need make no wonder of theſe being omitted. Beſides, where the place of the colure is too generally defined, and a ſtar cannot be found to direct it with ſufficient exactneſs; ſuch *data* as theſe ought to be left out. For any body that knows what he is about, will chooſe clear *data* before ambiguous ones; while ſuch a judge as this Doctor muſt take all without diſtinction. In order to determine this point, Sir I. Newton, with uncommon ſagacity and judgment, has in either caſe, choſen *five* of the beſt defined, and moſt unexceptionable places, to found his calculations upon; which he has compleated with as much ſkill and nicety, as the ſolution of ſuch a problem deſerved. So that the method he has followed will be admired and approved by all proper judges; tho' it be condemned by ſuch ignorant, perverſe critics as this, that are too dull to ſee the reaſon and beauty of ſuch a proceſs.

By this judicious way of proceeding, he has found the mean places of theſe colures (from theſe five), to be *Taurus* $6^s\ 29^\circ\ 15'$, and *Leo* $6^s\ 28^\circ\ 46'$, which are ſo ſurpriſingly near, that the moſt ſcrupulous cannot expect any thing nearer; which is a full teſtimony of its truth. So that at a mean, the four cardinal points had gone back $1^s\ 6^\circ\ 29'$, which amounts to 2627 years. Nay further than that, thoſe very places which this Profeſſor ſays he left out, he ſhews that they all fall in the ſolſtitial colure, as *Eudoxus* had deſcribed them; and conſequently that this had all the characters of that colure, according to that deſcription. But this purblind critic either could not or would not ſee all this. This being all made plain, will he ſay that *Eudoxus* lived ſo many years ſince? Yet he ſays as much, when he ſays *Eudoxus* deſcribed the ſphere as it was in his own time. But as far as I can ſee, he will neither allow the colures to paſs thro' the middle of the ſigns in *Eudoxus*'s time, nor *Chiron*'s time, nor any time elſe; ſo ſtrangely full of contradictions is he.

What he ſays about the little Bear is nothing to the purpoſe. If it was not formed into a conſtellation in *Chiron*'s time, what had Sir I. Newton to do with it. And even if it was, it was too near the pole to make any calculations upon; but any way, *Eudoxus* could not deſcribe the path of the colure in other words. But if the Profeſſor would have done any thing to the purpoſe, he ſhould have calculated how far this ſtar was from the colure; and then a better judgment could be made. He likewiſe ſays, *it is impoſſible to conclude from this deſcription of the cardinal points, that Chiron placed them in the middle of the conſtellations.* Now had the Doctor made any calculations to ſhew this impoſſibility, or to ſhew how far any of theſe points deviate from the colure, he had done ſomething. But he ſeems to be better qualified for forging arguments, than making calculations. But Sir Iſaac has actually made calculations, and has found and demonſtrated all theſe deſcriptions to be right, by the ſurprizing agreement among all theſe obſervations; which is ſufficient to overthrow all the Doctor's wild gueſſes, and ſilence all his cavils at once. And indeed it is no wonder to find him miſtaken in a matter, which is quite out of his ſphere; when he miſtook in a matter within his own ſphere, about the nature of *Virtue*; till a Lady, Mrs. *Coburn*, ſet him right. *O Doctor!*

3. We come now to his third grand article, *whether they knew how to draw the colures exactly.* This is ſuch a childiſh objection, that I don't know what anſwer is fit to be given to it. If he means perfectly exact, there is no man at this day can do it.

For at this rate the colures are to be drawn preciſely thro' ſuch and ſuch ſtars, which can be looked on no otherwiſe, than as ſo many mathematical points, which cannot be. All that can be concluded from hence is, that each colure, when rightly drawn, will paſs ſo near any ſtar, that the difference cannot well be diſcovered, by ſuch inſtruments as were then in uſe; or that ſuch and ſuch remarkable ſtars, come the neareſt thereto of any. But let us ſee how he argues upon this point. He tells us, *Eudoxus was deſcribing the ſphere as it ſtood in his time*; and in the very next words he confutes himſelf by telling us, *we are ſure that he placed them wrong, for we can prove that they were not in the middle of the conſtellations in his time.* Was ever poor Doctor ſo embarraſſed with an argument; he firſt tells us, that *Eudoxus* was deſcribing the ſphere in his own time; and immediately after, that ſuch a deſcription did not at all agree with the ſphere at that time. Would not any body but this Objector then conclude, that it was not deſcribed for that time, but for ſome other. Every circumſtance leads him, as it were, by the noſe, into a direct path, and yet he moſt ſtrangely miſſes his way. Inſtead of doing that, he has nothing to recur to, but that ridiculous ſhift, that *Eudoxus did not know how to draw the colures.* Here then it ſeems we have a famous Aſtronomer, and the greateſt of the age, that did not know how to draw one great circle perpendicular to another on the ſphere, he was ſo very ignorant. I know not what this Doctor may think, but I believe all the world will laugh at ſo idle a chimera. But the Doctor keeps fluttering about among theſe abſurdities, not knowing where to fix; but will readily agree to any impoſſible ſcheme, rather than to Sir *I. Newton.* Here one would think, if *Eudoxus* had miſſed his aim in drawing thro' one ſtar, he would ſcarce have miſſed two; and if he miſſed two, he would hardly miſs three; but by ſtrange ill luck, he has *miſſed* them all, according to him. And yet Sir Iſaac has ſhewn that he has *hit* them all. So that this great Profeſſor has made a fool of *Eudoxus*, or elſe of himſelf. But Profeſſor *Simſon*, of *Glaſgow*, gives a quite different account of *Eudoxus*; he tells us, that it is the opinion of ſome learned men, *that Eudoxus was the author of the chief propoſitions in the 12th book of Euclid.* See his notes on B. 12. Euclid.

But to ſhew you how great an Aſtronomer this Profeſſor is, he quotes *Hipparchus* for ſaying, that no ſecondary of the equator, could paſs thro' all the places, before mentioned to be, in the colures. Now if this Gentleman knew any thing of the ſphere, he would not at all wonder at this; becauſe it is impoſſible, that the two great circles (the two colures) which were perpendicular to the equator in the time of *Chiron*; I ſay, it is impoſſible that the ſame two great circles ſhould be perpendicular (or ſecondaries to) the equator, in the time of *Eudoxus*: For *Hipparchus* knew better. And this is another proof, that the ſphere deſcribed was not for the time of *Eudoxus*, but for ſome other preceding time. And hence this Rev. Doctor need take no further pains to make *Eudoxus* an ignoramus, for it will all fall upon himſelf; all his arguments being founded upon his own ignorance; and all the conſequences he draws from his abſurd reaſoning, muſt all fall at once. But where is his modeſty, thus to fall foul of a perſon of ſuch tranſcendent abilities as Sir Iſaac; and who has obliged the world with ſo many eminent diſcoveries, and which were thought before to be beyond human art; and yet with no better ſucceſs, as he himſelf appears to be no judge of it; and with no better deſign than to degrade that great man, who ſhines brighter for being cavilled at. For the ſame ſpirit and genius that runs thro' his other noble works, are equally viſible in this of his *Chronology*.

I think it needleſs to purſue this learned Doctor any further; it is plain he did not underſtand what he was writing about: every incident confutes him. But Sir Iſaac ſtands firm, his plan is confirmed not only by the agreement among the obſervations, but alſo by a great many other proofs of a different nature, all conſpiring to prove the ſame thing. And in ſuch caſes as this, which are not capable of mathematical demonſtration; the concurrence of ſo many circumſtances muſt make the thing ſo highly probable, as to take away all doubt and ſcruple, and give entire ſatisfaction.

After it was known that Sir *I. Newton* had writ a book of *Chronology*, a foreigner (one *Abbe Conti*) under the pretence of friendſhip to Sir Iſaac, obtained a copy of the Chronicle or Index, with a ſolemn promiſe to keep it ſecret. But obſerve how he kept his promiſe; he communicated it to one *Souciet*, a jeſuit, who got a bookſeller at *Paris* to print it, without Sir Iſaac's knowledge or conſent; and with it a pretended confutation of the Chronicle, by the ſaid *Souciet*. This was ſent to Sir Iſaac, who confuted every article of this pretended confutation. So that this may be ſaid to be ſtolen from him, much after the manner that Leibnitz ſtole his Fluxions; but with a worſe deſign, to involve Sir Iſaac in diſputes to oblige *Leibnitz*.

And the ſame *Souciet*, after Sir Iſaac's death, writ another Tract againſt him. But Dr. Halley ſhewed him, that thro' his ignorance in Aſtronomy, he had made falſe calculations; and had aſſumed a wrong ſtar, for the firſt ſtar of Aries; and ſhewed him when all theſe things were rectified, the matter would be juſt as Sir *I. Newton* had deſcribed it. Thus you ſee how this great man has been uſed.

Of Dr. Bedford's Chronology.

THIS Author is one of the ſame ſtamp as the foregoing, or rather more enthuſiaſtical; for if you will believe him, he can tell the very day when the moſt remarkable events in the beginning of the world happened. He promiſes a large book with maps, wherein, no doubt, he will inform you of the very day when the world began, where the garden of *Eden* was placed, where the Ark reſted, and how every Spot of Ground was peopled after the Flood, &c. And all ſuch difficulties as are without the power of Sir *I. Newton*, or any body elſe beſides. For in the Advertiſement to the preſent book, he tells us, that the ſun was created in the Equinox on a *Thurſday*, and the moon at full; that God ſpoke to *Noah* on a *Sunday*; that *Noah* ceaſed from bringing the living creatures into the Ark, on a *Sunday*; that God ſpoke to him to come out of the Ark on a *Sunday*; that God ſpake to *Abraham* on a *Sunday*; that *Solomon* dedicated the temple on a *Sunday*; that the angels ſung Glory to God, for the birth of Chriſt, on a *Sunday*, &c. &c. So that our Author, to be able to ſolve ſuch problems as theſe, without any *Data*, cannot be leſs than inſpired; and ſo I believe he is, with preſumption and vanity. But I think few people, who are thus glutted with ſuch impoſſibilities, will think it worth their while to penetrate any further into this profound Treatiſe; in which he is as confident, as if he had been preſent at every event. I ſhall therefore only give a few ſpecimens of his fine reaſoning.

He begins his book with a moſt fulſome *Encomium* upon Sir *I. Newton*, as a great Mathematician; tho' he was no manner of judge about it, as will be ſeen after. He tells you, that Sir Iſaac's Chronology was writ in his dotage; tho' Sir Iſaac expreſsly ſays, he did it when at *Cambridge*, and therefore when he was a young man; and he only reviſed it when he was old. And in reality it had been the work of his whole life-time, at his vacant hours. This Author tells us, the reaſon that Sir Iſaac would not publiſh it in his life-time was, that he was conſcious of his miſtakes and errors. And he may as well ſay, that his refuſing to print his *Optical lectures*, or his book of *Optics*, was becauſe he was conſcious of their being erroneous. So ready are ſuch authors as this to give falſe reaſons for things, and miſs the true; for the true reaſon was, to prevent waſting his time in uſeleſs diſputes, with weak and obſtinate perſons, not worth a diſpute, like this Author.

This Doctor ſeems to be under great concern, that Sir Iſaac's Chronology ſhould differ from the Scripture Chronology; but I believe it is rather for differing from three or four biſhops he mentions. For Sir Iſaac takes care all along to connect his account with that of the Scripture; tho' perhaps not according to the ſenſe of this Author. And becauſe Sir Iſaac differs from all other Chronologers, therefore he thinks it muſt needs be falſe, and ſo he condemns it at once, becauſe not agreeable with him and his four Biſhops.

It is no news to tell us, that Sir *I. Newton*'s Chronology differs from all the world; it was Sir Iſaac's profeſſed deſign. He attempted to correct the antient Chronology, which no body elſe could do, or at leaſt has done. But this Rev. Doctor gives us nothing but a *hotch potch* of the common Chronology, with all its inconſiſtencies and errors; which he knew not how to get rid on, or to mend. This he intermixes with his own wild conjectures;

and a romantic account of his own, taken at random from all ſorts of authors. But let us ſee how he goes on.

Arguing againſt Sir *I. Newton* about ſhip building. He tells us, that *the firſt pattern for ſhips was the ark, which was made ſquare, that it was not deſigned for ſailing, but to lie on the water.* How comes it then to be a fit pattern for ſhips, that are to ſail from one place to another? And he tells us, that *tall and long ſhips were probably invented by Ham, who having been in the ark, could not be ignorant of navigation.* As if a man that was cloſe ſhut up in a trunk, which was to lye upon the ſea, could from that ſituation, conceive any notion of navigation: ſo little knows he what *Navigation* means.

Sir *Iſaac* ſays, the Chronology of things done in Europe above 80 or 100 years before *Cadmus* (who invented letters) cannot be admitted. This Doctor replies, *and the hiſtory of Moſes, of the things done above* 100 *years before his time, will be deſtroyed by the ſame argument.* Here he puts *Moſes* upon the ſame footing as other common hiſtorians, he owns no difference of circumſtances.

Sir Iſaac tells us, that *Seſac* left geographical tables of all his conqueſts at *Cholchos*: and there *Geography* had its riſe. This Objector has nothing to anſwer but, *it is very ſeldom that ſuch arts are found out by martial men, who have ſomething elſe to do.* Here is another inſtance of his ignorance; for can any man invade a country, but he muſt know or find out the ſituation of places, how they are bounded by one another, or by the ſea, &c. And the knowledge of this is *Geography*. And his ignorance is ſtill further confirmed in this, where he makes *Sihor* not to be the *Nile*, but a river in the country of the Amalekites, where there is no ſuch river. When he writ this, he certainly was *not capable of thinking.*

Sir Iſaac ſays, the temple built by *Menes* could not be above two or three hundred years older than *Pſammiticus* that finiſhed it. This Doctor anſwers, *Chronology may be reduced ſtill further; for* Herod *the great, who was alive at the birth of Chriſt, finiſhed* Solomon's *temple; and therefore it could not be above two or three hundred years older than the reign of* Herod. So that this blundering writer cannot tell the difference between *finiſhing* and *repairing*. There are endleſs examples of ſuch wretched ſtuff as this given for reaſons and arguments, and hardly any thing better.

His repreſenting Sir *I. Newton as not capable of knowing what was demonſtration and what was not*; is very conformable to the ſentiments and diſpoſition of ſuch a brute as this, that has not the leaſt idea of the nature of a demonſtration. Now for ſome ſtrokes of his aſtronomical genius.

The firſt thing of this ſort that he preſents us with, is the calculation of the ſun's place 2066 years before Chriſt, which place he falſely makes in *Capricorn* 14° 8′ 19″; which ſhould be *Sagitary* 13° 23′ 20″. The Egyptian year did not take place 1319 years before *Nabonaſſar*; and if it did, this abſurd calculation proves nothing at all. Yet from this, he makes the diſtance of the ſun from the tropic only 45′, when he is 21 days from it, by his own calculation. Nor does it appear by what ſort of legerdemain, he makes 45^m anſwer to 21 days. Alſo it may be obſerved, from his proceſs, that he makes the Julian year take place from the Era of *Nabonaſſar*; which is another blunder.

When the Julian year begun, he tells us, the beginning of the year was, when the ſun was in Capricorn 8° 50′ 57″, and its diſtance from the tropic 18′; ſo by this, the tropic was in about 8° 33′ of Capricorn, which is an abſurdity.

He makes a remark from *Scaliger*, that the colure could not paſs thro' the tail of the ſerpent, altho' there is no calculation made about the ſerpent; ſo that this is nothing to the purpoſe. Yet he concludes that the colure had been placed wrong, juſt the ſame as Dr. *Rutherforth* ſays, an aſtronomer as wiſe as himſelf.

This author tells us, *the mariners at ſea find out the latitude of the place, not from the exact height of the ſun at noon*; and from ſuch ſtuff as this, he tells you that *Chiron* (or ſome other) *would place the ſummer ſolſtice and its colure, not at the entrance of Cancer itſelf, but before it.*

In quoting *Hipparchus*, he tells us *Hipparchus well knew there was* 1100 *years between the time of the argonautic expedition, and the diſcovery of the motion of the equinox*. But certainly the author does not *well know* what he writes, for if *Hipparchus* had known this ſo well, he would have told us ſo. And all the conſequence he draws from this ſuppoſition is nothing but a heap of abſurd nonſenſe.

The next aſtronomical flouriſh he makes, is to calculate the time of *Arcturus*' riſing; but before he begins, he has his way to pave with ſo many *if's* and *and's*, and *ſuppoſitions*, that the reader muſt be ſurfeited before he begins his calculation. Then he aſſumes the lat. of *Cuma* (but it was *Aſcra* where *Heſiod* lived), to be 38° 54′, which ſhould be 39° 20′ at leaſt, but no body can tell exactly; but a ſmall error will do him a deal of ſervice, and he can apply it his own way. In calculating from his data, he finds the declination of *Arcturus* an. 870 before Chriſt, to be 34° 25′, which ſhould be 34° 19′ 32″; and his right aſcenſion 179° 32′, which ſhould be 178° 21′; and ſo finds *Arcturus* at ſun-ſet, 2° 56′ under the horizon, which ſhould be 1° 46′, by his own data, if he had wrought right. Now this eminent aſtronomer, or his computer,

puter, has made no manner of allowance for refraction; which being done, it will bring *Arcturus* very near the horizon, as it ought. Nor does he take notice of refraction in the reſt of his ſublime calculations that follow. But to make all clear, he tells us, that the number 60 may ſtand for 55, or 65, &c. which is a *poetica licentia*, that I did not underſtand before.

This doctor tells us Sir Iſaac's whole book is built upon this, that *Chiron* made a ſphere, &c. But this is a moſt impudent falſehood, for the whole book is not built upon this; it ſtands upon many more pillars, which this writer, with all his ſophiſtry, is not able to demoliſh. Whether *Chiron*, or whoever made the ſphere, it will be the ſame thing; if it was made at that expedition, or a little after, as there is no room to doubt.

This Rev. Doctor contends that the ages of men were longer in ancient times than at preſent; and therefore he thinks Sir Iſaac has made them too ſhort. But he cannot make it out, that they were any longer within the bounds of this chronicle. And *David* will be againſt him; for he tells us, not of one particular man's age, but in general, that the age of man is 70 or 80 years, as at preſent,

And the like objections he makes againſt the reigns of kings; tho' Sir Iſaac has confirmed the thing by ſo many catalogues of vaſt length. And this Author's picking out ſhort catalogues of greater lengths, whether of ages or generations, is nothing to the purpoſe. He may as well take one age or one generation, as a ſtandard of all the reſt. But this is not doing juſtice to the cauſe.

This worthy Doctor is ſo enraged, that Sir I. Newton's Chronology is oppoſite to all the world beſides; that he damns it at once by wholeſale; and falls into ſuch a ſtrain of raillery as is hardly to be matched in all Billingſgate. He can talk of nought for ſeveral pages together but *contradicting the Scriptures.* This ſyſtem of *Chronology* giving men a *diſmal tincture*, rooting-out of *religion*, bringing in *infidelity*, that ſuch *poiſon* ought not to go abroad; the *Devil's* ſpite againſt the *ſacred oracles*, and the *muſtering* of his *forces*, *blaſphemies*, *hereſies*, liberty for *new* religions, *Atheiſm*, *Deiſm*, *Hell torments*, man's *immoralities* and *debaucheries*, attacks on *Chriſtianity* and Scripture *Chronology*; *ſmoaks* ariſing out of the *bottomleſs pit*, that *darken the ſun and the air*, &c. &c. and, I believe, by the time that the reader has got thro' it all, he will be nearly choaked with all this ſmoke. And all this Rant is, becauſe men will not ſwallow every ſtupid thing, that ſuch men would cram down their throats. After ſo pathetic an harangue, no body need have any doubt about his pacific diſpoſition; this pious Doctor, I believe, will hardly ſend any man to heaven, that ſhould preſume to give any credit to ſuch a vile book as Sir *I. Newton*'s Chronology. But I underſtand not why all this ſtuff is introduced here into a book of Chronology; except this pious Doctor meant to call down *fire from heaven* upon all ſuch wicked perſons, as ſhould take the liberty to differ from him in *Chronological matters*, as well as in *Religion.*

He frequently mentions Sir *I. Newton* as a great and eminent Mathematician, nay, the greateſt in the world. Yet he dare not truſt him with the ſolution of certain aſtronomical problems; but attempts to give more exact ſolutions to them himſelf. At this rate he muſt be a greater mathematician, and a *greater* than the *greateſt* in the world. Or perhaps Aſtronomy muſt rather be reckoned a part of Divinity, than of Mathematicks.

However compleat he may be in the mathematical arts, he is certainly compleat in the art of

Squabbling. For example, Sir Iſaac tells us, that the obſervations of the ancients were but *coarſe*. This word is of infinite ſervice to him, and he *chews* it over and over. By the *magic* of this word he can make any error, great or ſmall, ſerve his turn. For if he miſſes his mark never ſo far, he can always account for it from the *coarſe* obſervations of the antients. This with him is a word of great latitude, and has no bounds.

I ſuſpect what irritates his temper moſt of all is, that this regular ſyſtem of Sir Iſaac's is likely to overturn his great *Babel*; which makes him labour with might and main to ſecure it; but with more *pains* than *prudence*. For there is as much difference between them, as between a regular and beautiful garden, and a confuſed wilderneſs.

But to take my leave of *this Author*, this renowned Champion for vulgar errors; Sir Iſaac having proved *Oſiris*, *Bacchus*, and *Seſoſtris* and *Seſac*, to be all one and the ſame perſon. As each of them was king of all *Egypt* at the very *ſame* time, and by a great many other circumſtances common to them all. This acute Reaſoner argues thus; *there were three famous men in the world*, *Hannibal*, *Belliſarius*, *and Churchil*; *they were all very eminent generals*, *remarkable for their courage and conduct*, *led their armies into foreign countries*, *fought many battles*, *took many towns*, *&c. therefore they are the ſame with the Duke of Marlborough*.

But pray ſtop a little Rev. Doctor, did all theſe men go into the *ſame* foreign countries, fight the *ſame* battles, take the *ſame* towns, &c? if not, where is the ſimilitude? Now, Sir, as ſoon as you can make out, that theſe four men were each a general of the *whole* army, at one and the *ſame* time, in one and the *ſame* place (as the parallel requires); then in ſpite of your *Logic*, they will be one and the *ſame* man, or rather four different names for the *ſame* man. This Doctor is certainly very unhappy in drawing parallel caſes, to chuſe ſuch where all the circumſtances are different, inſtead of being the ſame or like. But for all that, the concluſion will be the very ſame with him, whether the circumſtances be the ſame or different. Does not his ſcarlet bluſh at this! Could any body of common ſenſe offer ſuch childiſh things to the public. He certainly has a mean opinion of his readers, to think they can be thus impoſed on. Yet all his parallel caſes are of the ſame kind.

FINIS.

ERRATA.

Menſuration, p. 104, l. 2b. *read* $\frac{\cdot 7854}{3} \times \mathrm{CD}$, is the

Surveying, Preface, p. ii, l. 3b. 4b. *read* μέγα

ERRATA.

b ſignifies, *from the bottom.*

pag	line	read
5	2b	SA or TA;
6	0	pa. 6.
9	7b	$Sp^3 \times pv^3 =$
	6b	And $SP^3 \times PV^3 =$
10	1	round R, in P : :
	2b	rectangle *uPv*.]
	1b	$Qv^2 + uPv =$
11	2	$Pv \times uV + uPV = VPv$
	4b	*Conics Prop.* 24. *Ellipſis*) perpendicular SY
	3b	(ſee alſo Cor. 7.)
12	1	dele *fig.* 5.
	12	[Pr. 21.]three—lines. If
	21	Lem. 16.
	24	iu the ſame
13	6	fig. 7.
	18	PI : : ſine of
	22	tangent at A,
14	6	dele fig. 11.
	10	fig. 11.
	11b	And *kn* : *ga* : :
	3b	O*e* : OD : :
15		dele *fig.* 13.
16	1b	IH : HF : : (*i*X : XY
17	1	dele fig. 20.
19	3b	: : *s* : PM =
29	13b	*d* in the ſame
	2b	*de*, *fg* touch
	1b	from C on
30	3	of B in *d* : :
	14	thro' $\beta\gamma$
	16	A as $C\beta$
	24	: : $C\gamma - \gamma\delta$
87	1	AZ, C*z*
105	15	$I\mu$: : $p\sigma$:
109	16	$\frac{G - C}{G - \gamma} Q =$
Pl.	I	fig. 6. for u write n.
		fig.4, 11, 12, for e write c.
		fig. 13. draw the prick lines OB, OeD.
Pl.	III	fig. 29. for n write w.
		fig. 36. for g write q.

John Quincy Adams.